

## Integration of cues

- Quick review of depth cues
- Cue combination: Minimum variance
- Cue combination: Bayesian
- Nonlinear cue combination: Causal models
- Statistical decision theory

## Distance, depth, and 3D shape cues

- Pictorial depth cues: familiar size, relative size, brightness, occlusion, shading and shadows, aerial/atmospheric perspective, linear perspective, height within image, texture gradient, contour
- Other static, monocular cues: accommodation, blur, [astigmatic blur, chromatic aberration]
- Motion cues: motion parallax, kinetic depth effect, dynamic occlusion
- Binocular cues: convergence, stereopsis/binocular disparity
- Cue combination

## Basic distinctions

- Types of depth cues
  - Monocular vs. binocular
  - Pictorial vs. movement
  - Physiological
- Depth cue information
  - What is the information?
  - How could one compute depth from it?
  - Do we compute depth from it?
  - What is learned: ordinal, relative, absolute depth, depth ambiguities

## Definitions

- Distance: Egocentric distance, distance from the observer to the object
- Depth: Relative distance, e.g., distance one object is in front of another or in front of a background
- Surface Orientation: Slant (how much) and tilt (which way)
- Shape: Intrinsic to an object, independent of viewpoint

## Distance, depth, and 3D shape cues

- Pictorial depth cues: familiar size, relative size, [brightness], occlusion, shading and shadows, aerial/atmospheric perspective, linear perspective, height within image, texture gradient, contour
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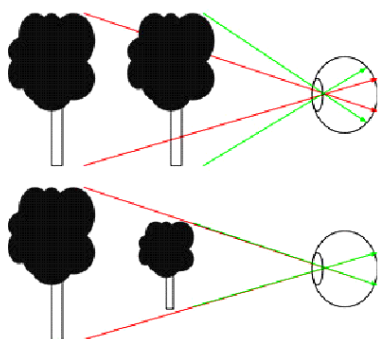
## Epstein (1965) familiar size experiment

How far away  
is the coin?



## Monocular depth cues

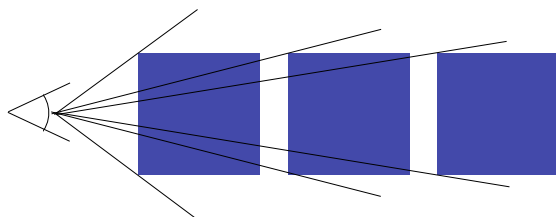
Retinal projection depends on size and distance



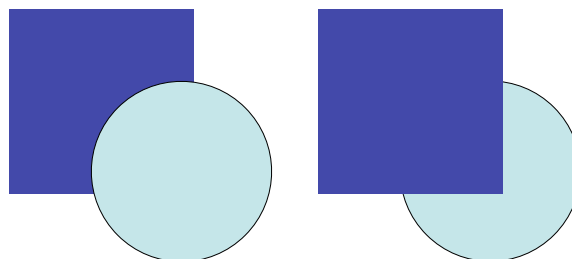
## Relative size as a cue to depth



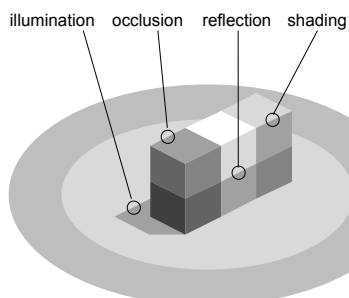
## Relative size as a cue to depth



## Occlusion as a cue to depth



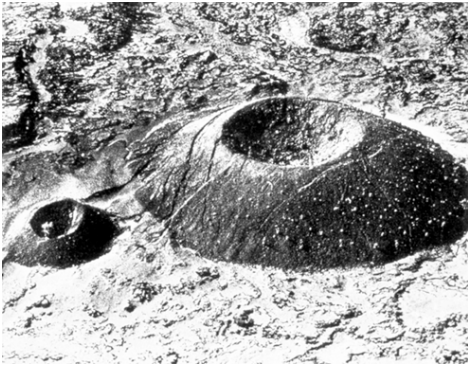
## Shading, reflection, and illumination



## Shading - prior of light-from-above



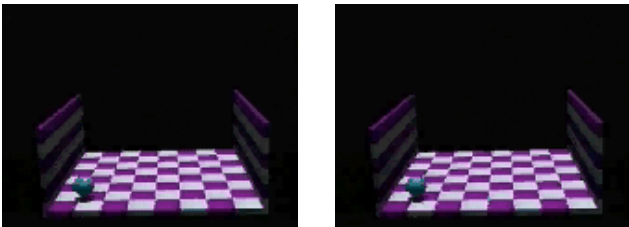
### Shading (flip the photo upside-down)



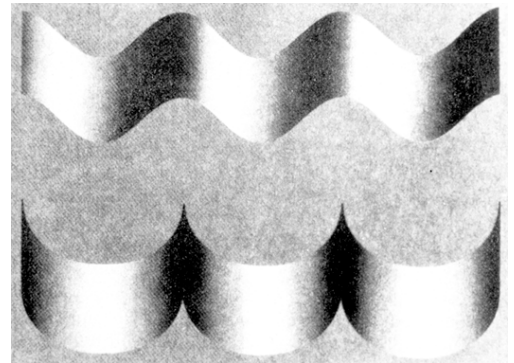
### Cast Shadows



### Dynamic Cast Shadows



### Shading and contour



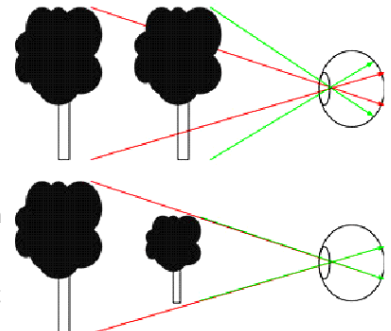
### Aerial/Atmospheric Perspective



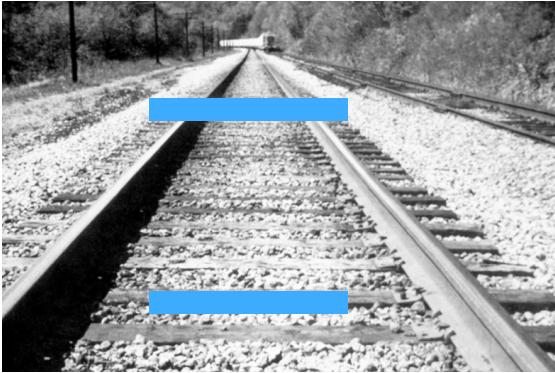
### Geometry of Linear Perspective

Retinal projection depends on size and distance:

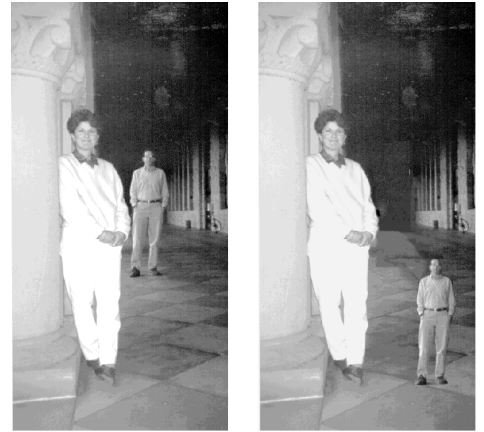
Size in the world (e.g., in meters) is proportional to size in the retinal image (in degrees) times the distance to the object



## Linear perspective

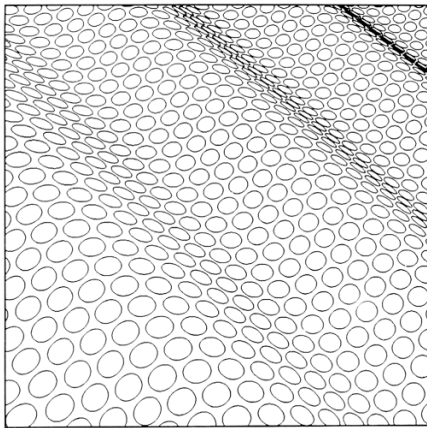


## Size constancy

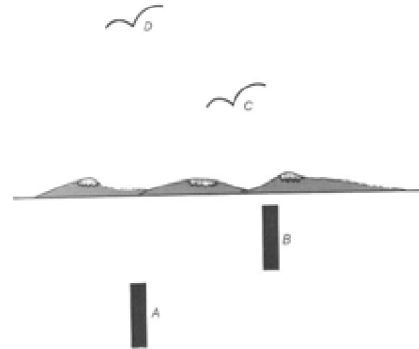


## Texture

1. Density
2. Foreshortening
3. Size



## Height Within the Image



## Distance, depth, and 3D shape cues

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## Monocular Physiological Cues

- Accommodation – estimate depth based on state of accommodation (lens shape) required to bring object into focus
- Blur – objects that are further or closer than the accommodative distance are increasingly blur
- Astigmatic blur
- Chromatic aberration



## Distance, depth, and 3D shape cues

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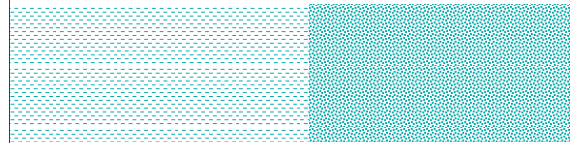
## Motion Parallax



## The Kinetic Depth Effect



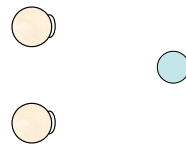
## Dynamic (Kinetic) Occlusion



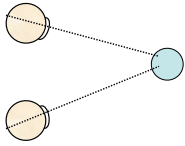
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## Vergence Angle As One Binocular Source



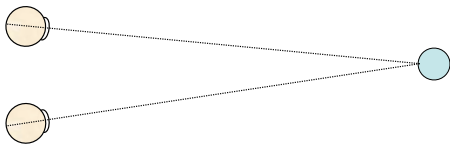
## Vergence Angle As One Binocular Source



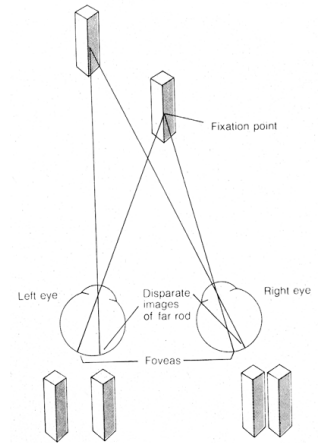
## Vergence Angle As One Binocular Source



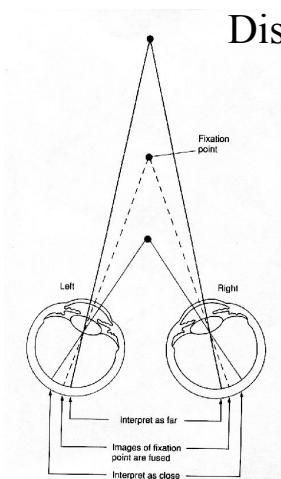
## Vergence Angle As One Binocular Source



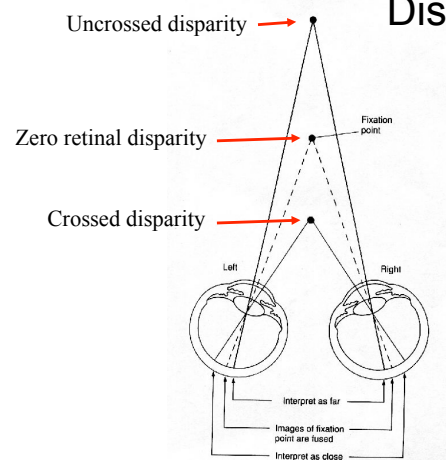
## Binocular disparity



## Disparity



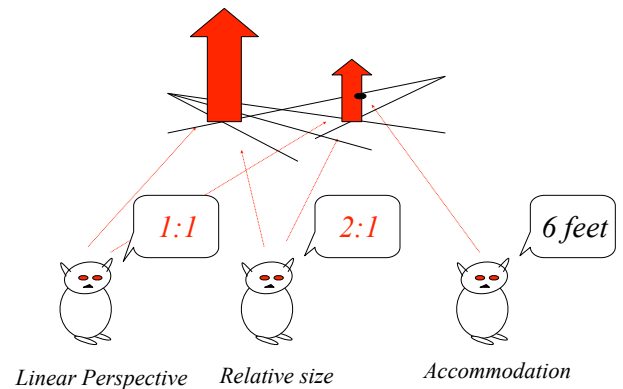
## Disparity



## Depth Cue Combination: Issues

1. How do you put all of the depth cue information together?
2. What do you do when cues disagree?  
A little ... ?  
A lot ... ? errors
3. How much weight should each cue get?

## When cues disagree ...



## Information Fusion Problem

Multiple sources of information,  
possibly in error, possibly  
contradictory

How combine the information into  
a single judgment?



Rashomon

## Optimal Cue Combination: Minimum Variance

$$E(X_1) = \mu_1, \quad E(X_2) = \mu_2$$

$$\text{Variances: } \sigma_2^2 \leq \sigma_1^2 \quad \text{Just use one cue?}$$

Suppose we use a linear cue-combination rule:

$$X = w_1 X_1 + w_2 X_2 \quad \text{weighted linear combination}$$

$$E[X] = w_1 E[X_1] + w_2 E[X_2] = (w_1 + w_2) \mu$$

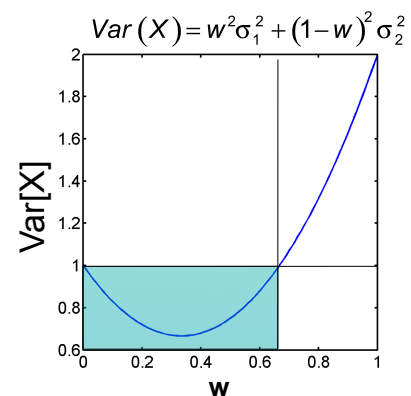
unbiased?

## Minimum-Variance Cue Combination

$$X = w X_1 + (1-w) X_2 \quad \text{unbiased}$$

$$\text{Var}(X) = w^2 \text{Var}(X_1) + (1-w)^2 \text{Var}(X_2)$$

$$= w^2 \sigma_1^2 + (1-w)^2 \sigma_2^2 \quad \text{minimize}$$



## Minimum-Variance Cue Combination

$$X = wX_1 + (1-w)X_2$$

$$\text{Var}(X) = w^2 \text{Var}(X_1) + (1-w)^2 \text{Var}(X_2)$$

Choose  $w$  to minimize variance:

$$w = \frac{1/\sigma_1^2}{1/\sigma_1^2 + 1/\sigma_2^2}$$

## Reparameterization

Define reliability  $r_i = \sigma_i^{-2}$

$$X = w_1 X_1 + w_2 X_2$$

weight proportional to reliability

$$w = \frac{1/\sigma_1^2}{1/\sigma_1^2 + 1/\sigma_2^2} = \frac{r_1}{r_1 + r_2}$$

$$r = r_1 + r_2$$

reliabilities add

## Cue Combination for Estimation

- Weighted average:

$$D(x, y) = \alpha_s D_s(x, y) + \alpha_m D_m(x, y) + \alpha_t D_t(x, y) + \dots$$

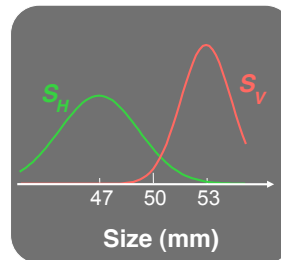
where

$$\sum_i \alpha_i = 1$$

- Optimal weights for independent cues:

$$\alpha_i = \frac{1/\sigma_i^2}{\sum_j 1/\sigma_j^2} = \frac{r_i}{\sum_j r_j}$$

## Combining Sensory Estimates

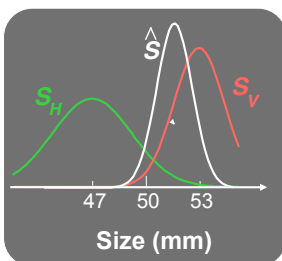


$$\hat{S} = w_H \hat{S}_H + w_V \hat{S}_V$$

$$w_H = \frac{r_H}{r_H + r_V}$$

$$w_V = \frac{r_V}{r_H + r_V}$$

## Combining Sensory Estimates

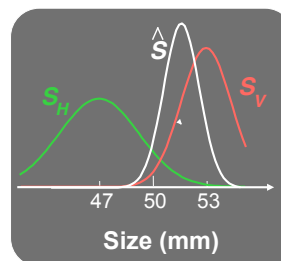


$$\hat{S} = w_H \hat{S}_H + w_V \hat{S}_V$$

$$w_H = \frac{r_H}{r_H + r_V}$$

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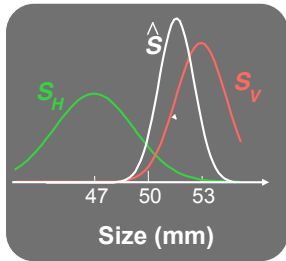
## Combining Sensory Estimates



$$\hat{S} = w_H \hat{S}_H + w_V \hat{S}_V$$

$$\sigma_{HV}^2 = \frac{\sigma_H^2 \sigma_V^2}{\sigma_H^2 + \sigma_V^2}$$

## Combining Sensory Estimates



Variance of combined estimate lower than variance of either single-cue estimate

$$\hat{S} = w_H \hat{S}_H + w_V \hat{S}_V$$

$$\sigma_{HV}^2 = \frac{\sigma_H^2 \sigma_V^2}{\sigma_H^2 + \sigma_V^2}$$

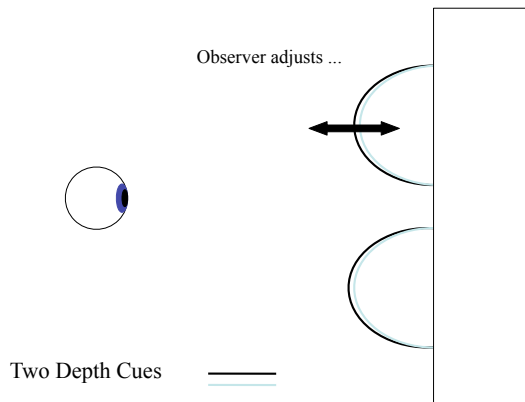
$$r_{HV} = r_H + r_V$$

## Perturbation Methodology and Influence Measures

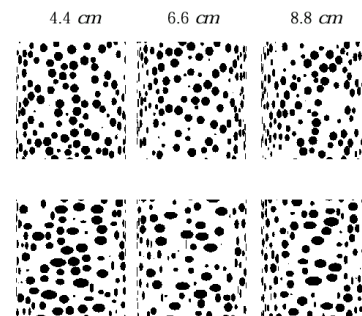
How can we measure the influence of various cues on perceptual judgments in complex scenes?

Goal: Change the stimulus as little as we possibly can.

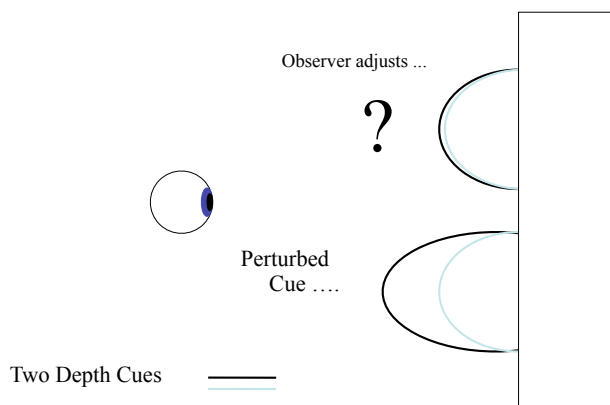
## Perturbation Method



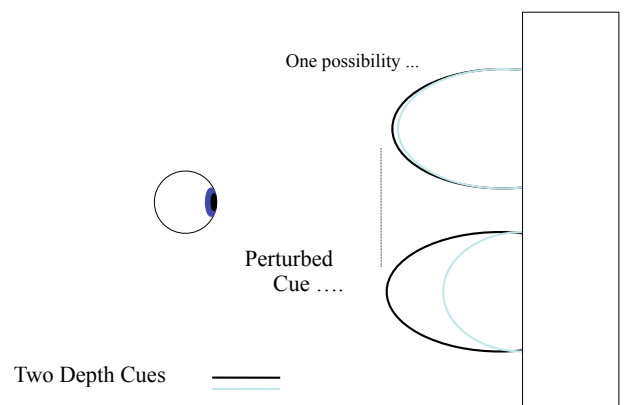
## Example: Texture and Motion



## Perturbation Method



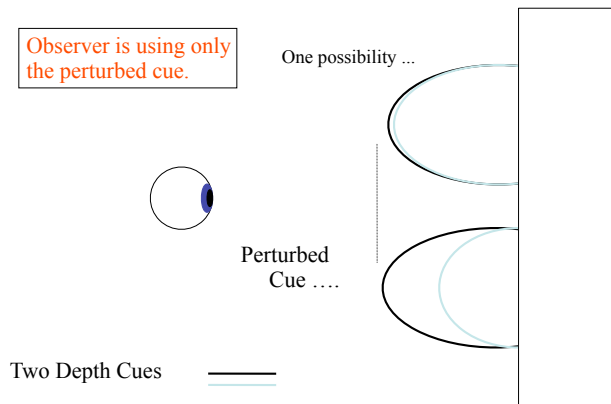
## Perturbation Method





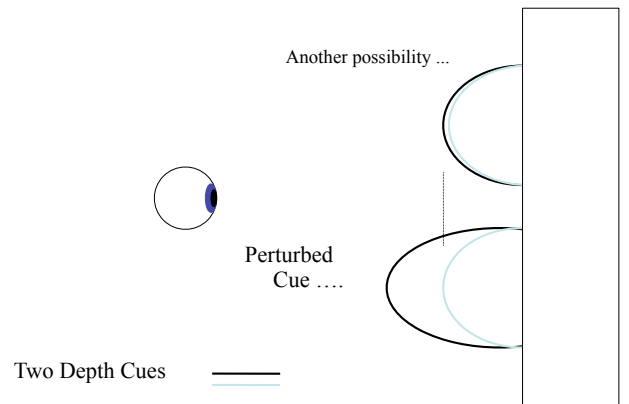
## Perturbation Method

Observer is using only the perturbed cue.



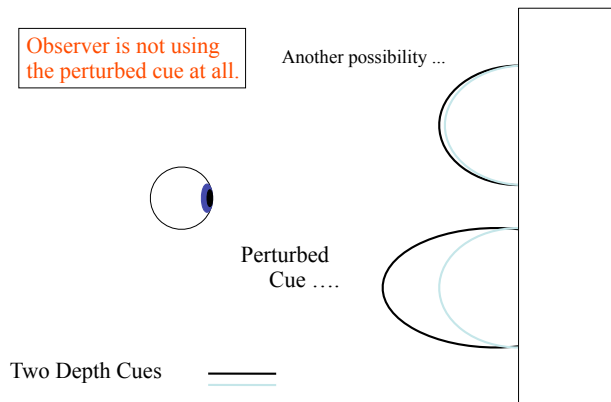
## Perturbation Method

Another possibility ...



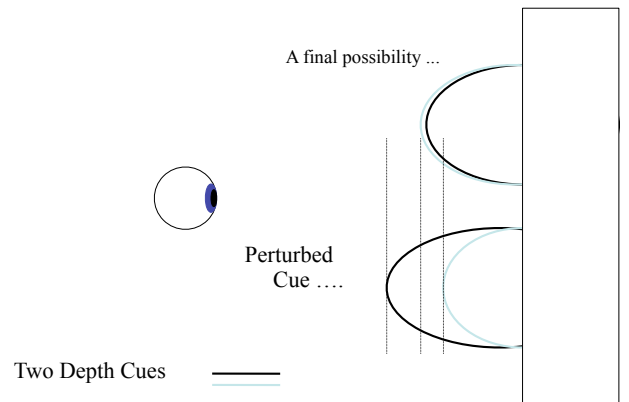
## Perturbation Method

Observer is not using the perturbed cue at all.



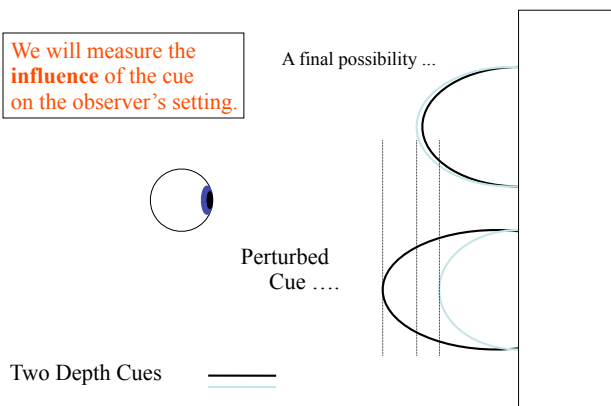
## Perturbation Method

A final possibility ...



## Perturbation Method

We will measure the influence of the cue on the observer's setting.



## An Experimental Paradigm: Perturbation Analysis

The observer's cue weights can be estimated.

The stimulus comparison:

$$Cue_1 = d \quad Cue_2 = d$$

Matches

$$Cue_1 = d_1 \quad Cue_2 = d_2 = d_1 + \Delta d$$

Therefore

$$\alpha_1 = \frac{d - d_1}{d_2 - d_1} = \frac{\Delta \text{depth}}{\Delta \text{cue}}$$

## Influence Measures

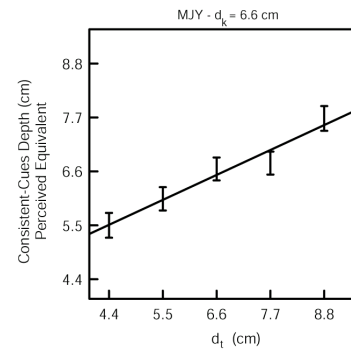
$$I_{cue} = \frac{\Delta_{setting}}{\Delta_{cue}}$$

*Influence of the cue* (pointing to  $I_{cue}$ )

*Change in observer's setting* (pointing to  $\Delta_{setting}$ )

*Perturbation of the cue* (pointing to  $\Delta_{cue}$ )

## Texture and Motion: Data



### Optimal Cue Combination: Bayesian

Compute posterior:

$$p(\text{depth} | x_1, x_2) = \frac{p(x_1, x_2 | \text{depth})p(\text{depth})}{p(x_1, x_2)}$$

Assume conditional independence:

$$p(\text{depth} | x_1, x_2) \propto p(x_1 | \text{depth})p(x_2 | \text{depth})p(\text{depth})$$

If likelihoods and prior are Gaussian, so is posterior, and means and reliabilities are as in minimum-variance case. Prior acts like a static cue.

### Optimal Cue Combination: Bayesian

$$p(\text{depth} | x_1, x_2) \propto p(x_1 | \text{depth})p(x_2 | \text{depth})p(\text{depth})$$

Depending on cost function and priors, choose:

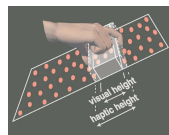
- ML: Maximum-likelihood estimator
- MAP: Maximum a posteriori estimator
- Mean of the posterior
- Median of the posterior
- Etc.

## Optimal Cue Combination

**Humans integrate visual and haptic information in a statistically optimal fashion**

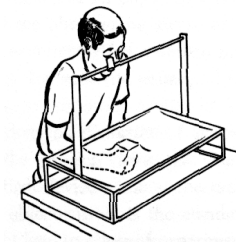
Marc O. Ernst\* & Martin S. Banks

Vision Science Program/School of Optometry, University of California, Berkeley  
94720-2020, USA



## Rock & Victor (1964)

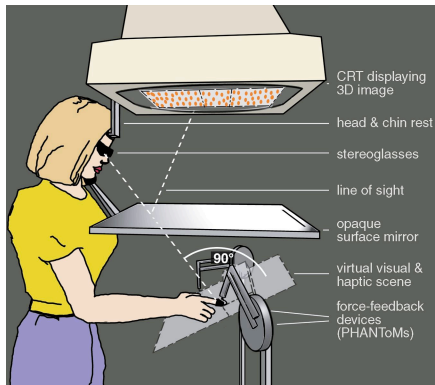
View object through distorting lens while exploring object haptically



*Visual capture*

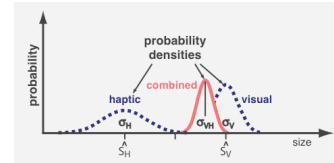
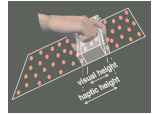
Visually and haptically specified shapes differed.  
What shape is perceived?

## Visual/Haptic Setup



## Visual Capture ?

Why should vision be the “gold standard” all other modalities are compared to?



$$S_{VH} = w_V S_V + w_H S_H$$

Weights

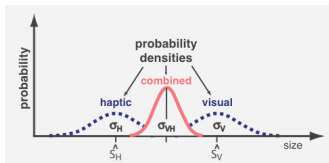
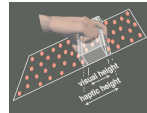
$$w_V = \frac{\sigma_H^2}{\sigma_V^2 + \sigma_H^2}$$

Variance

$$\frac{1}{\sigma_{VH}^2} = \frac{1}{\sigma_V^2} + \frac{1}{\sigma_H^2}$$

## Visual Capture ?

Why should vision be the “gold standard” all other modalities are compared to?



$$S_{VH} = w_V S_V + w_H S_H$$

Weights

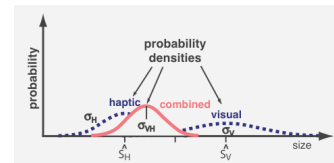
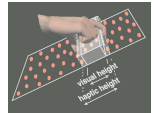
$$w_V = \frac{\sigma_H^2}{\sigma_V^2 + \sigma_H^2}$$

Variance

$$\frac{1}{\sigma_{VH}^2} = \frac{1}{\sigma_V^2} + \frac{1}{\sigma_H^2}$$

## Visual Capture ?

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$$S_{VH} = w_V S_V + w_H S_H$$

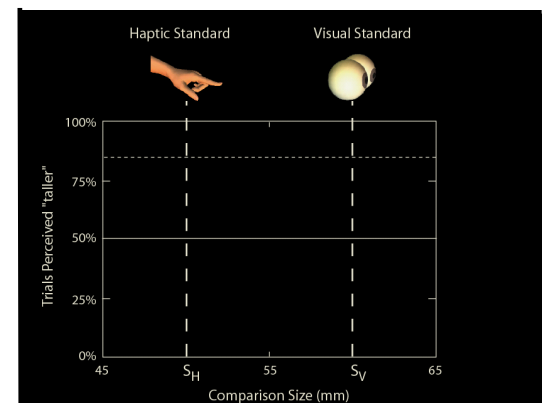
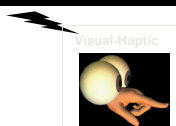
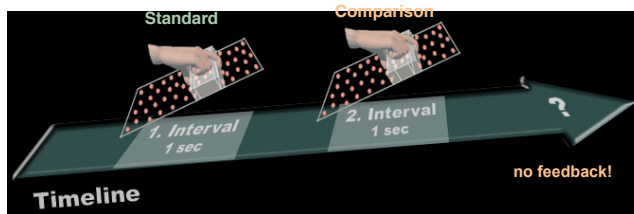
Weights

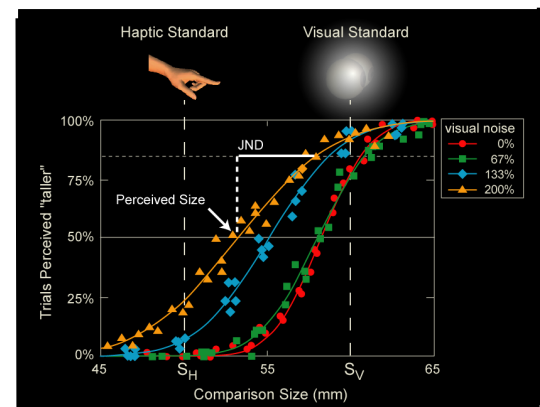
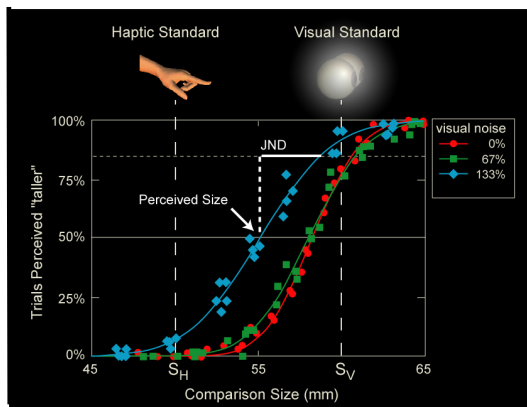
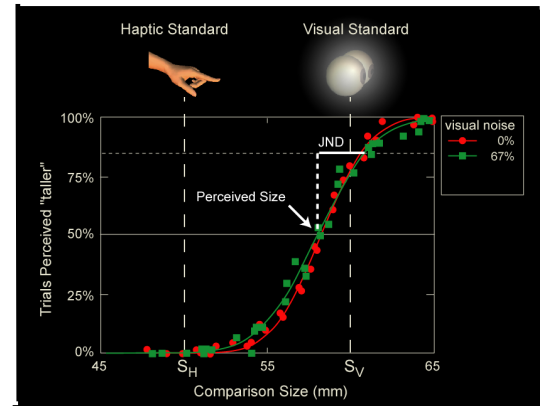
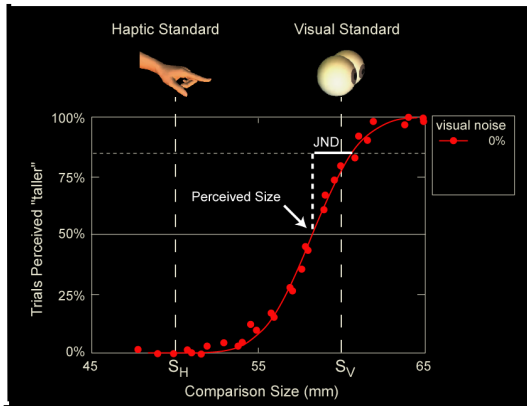
$$w_V = \frac{\sigma_H^2}{\sigma_V^2 + \sigma_H^2}$$

Variance

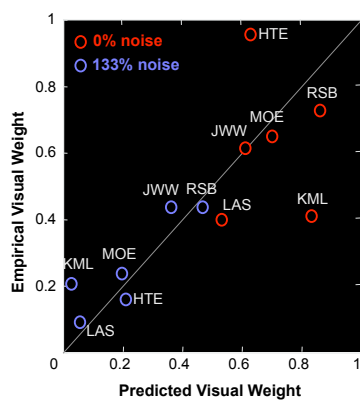
$$\frac{1}{\sigma_{VH}^2} = \frac{1}{\sigma_V^2} + \frac{1}{\sigma_H^2}$$

## 2-IFC Task

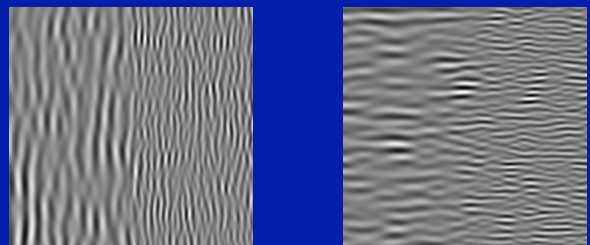




## Individual Differences

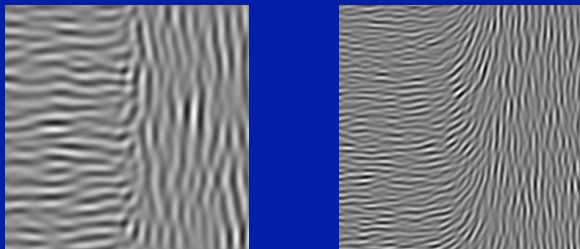


## Cue 1: Spatial Frequency



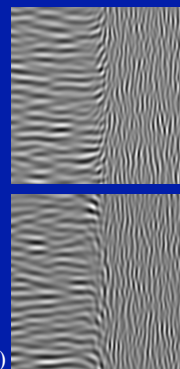
Landy & Kojima (2001)

## Cue 2: Orientation



Landy & Kojima (2001)

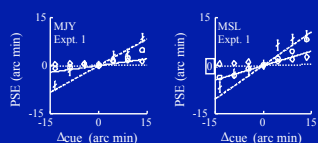
## Task: Vernier



Landy & Kojima (2001)

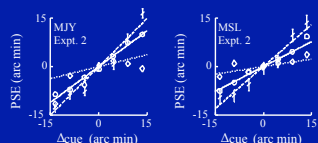
## Texture: Results

- —  $\sigma_o = 9$  arc min,  $\sigma_f$  or  $\sigma_c = 9$  arc min
- $\sigma_o = 36$  arc min,  $\sigma_f$  or  $\sigma_c = 9$  arc min
- ◇ —  $\sigma_o = 9$  arc min,  $\sigma_f$  or  $\sigma_c = 36$  arc min



(a)

(b)

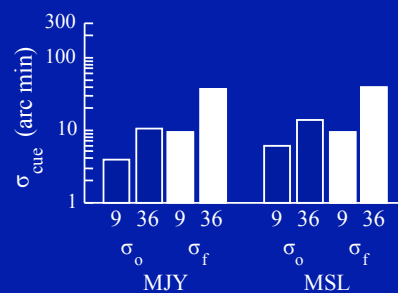


(c)

(d)

Landy & Kojima (2001)

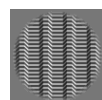
## Texture: Fitted Reliabilities



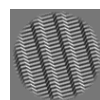
Landy & Kojima (2001)

## Demo: Landy/Kojima psychophysical task

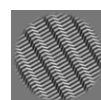
### Influence of priors: Mamassian & Landy (1998, 2001)



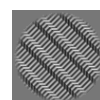
0 deg



15 deg

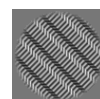
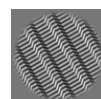
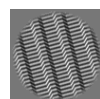
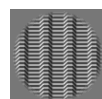


30 deg



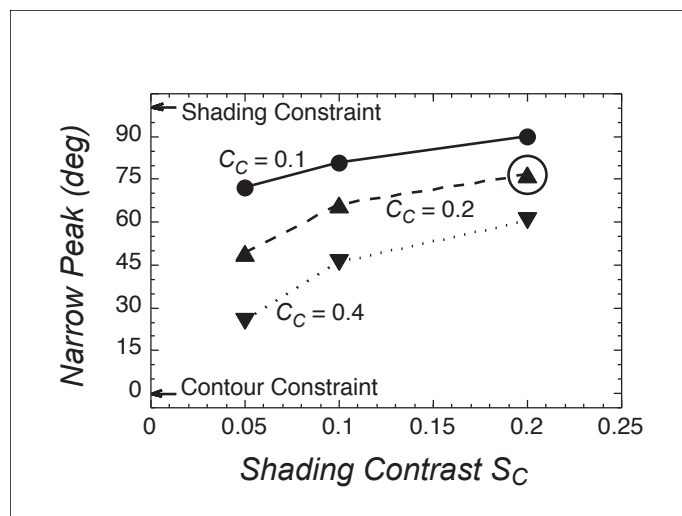
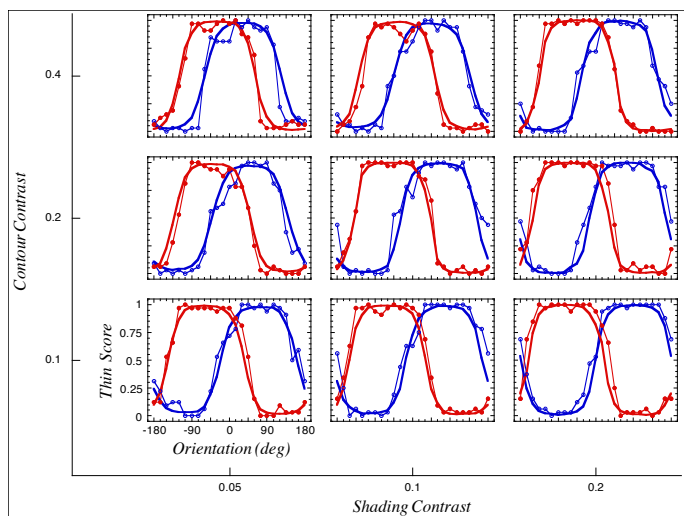
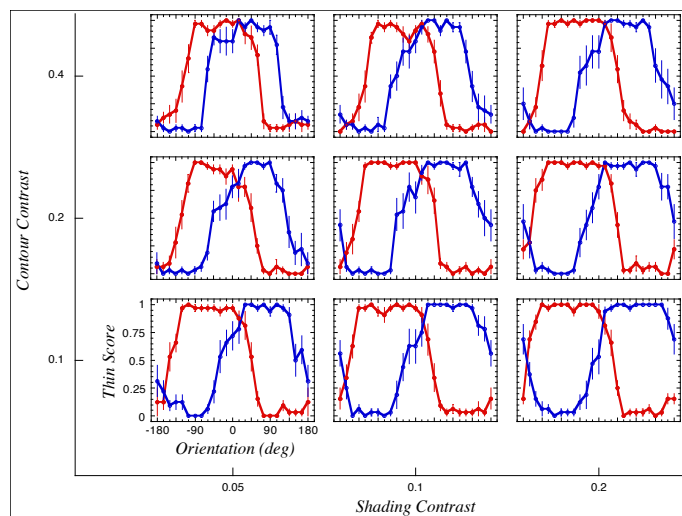
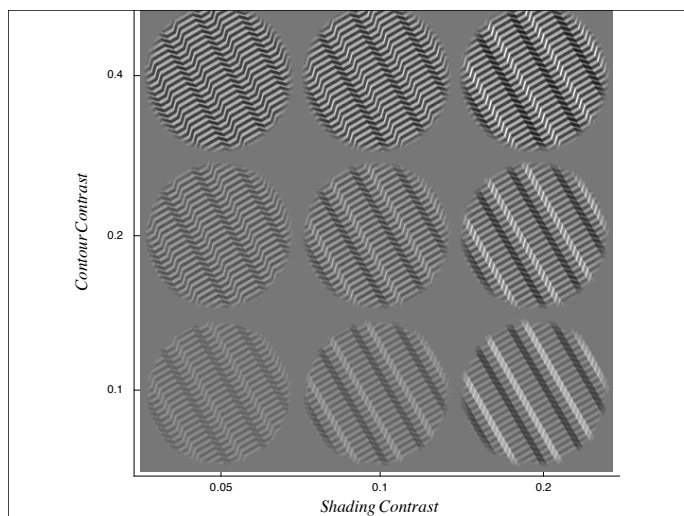
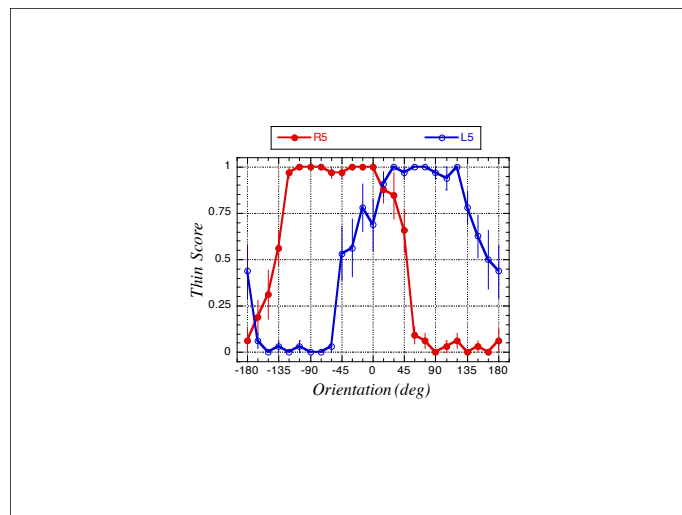
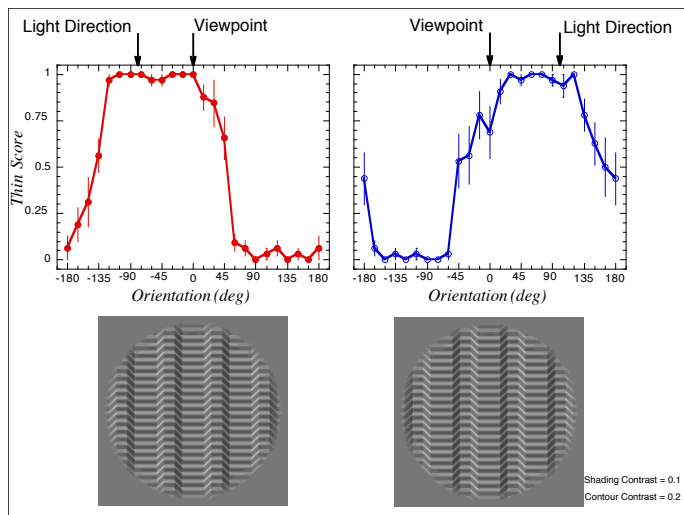
45 deg

...



...





## Cost functions

We've touched on two of the three elements of Bayesian estimation and Bayesian decision-making: the likelihood and the prior. But, what about the third element: the cost function?

## Typical Task for Decision-Making Under Risk

Would you rather have

A. \$480, or

B. A 50-50 chance for \$1,000?

A choice between "lotteries", where a lottery is a list of potential outcomes and their respective probabilities of occurrence, e.g.,

(0.5, \$0; 0.5, \$1,000)

## Typical Task for Decision-Making Under Risk

Would you rather have

A. \$480, or

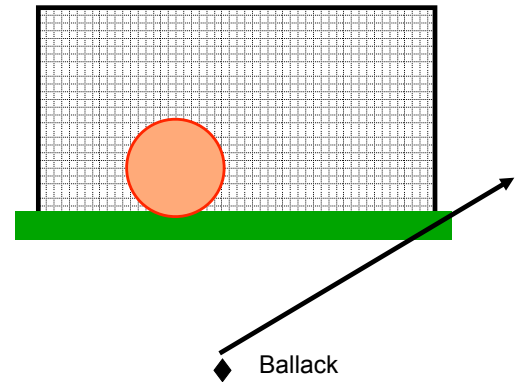
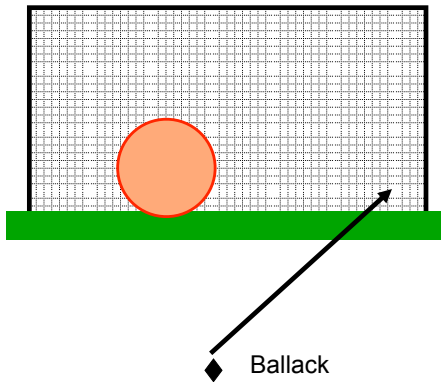
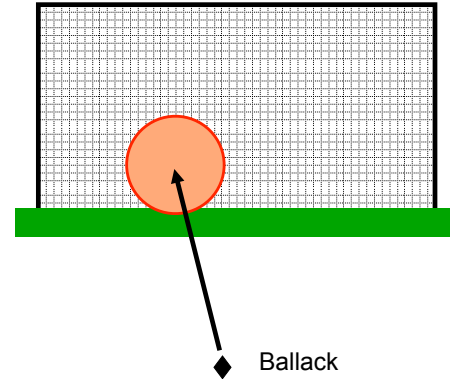
B. A 50-50 chance for \$1,000?

Typically, people choose A, showing risk-aversion for gains, and also show risk-seeking behavior for losses, along with many other "sub-optimal" behaviors, i.e., they don't simply maximize expected gain.

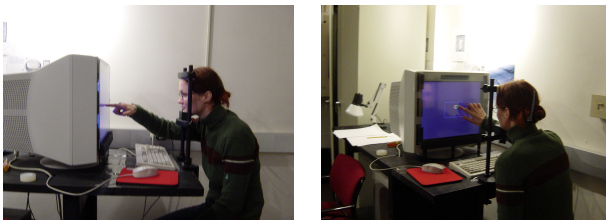
## An Implicit (Motor) Decision Task

World Cup 2002, semifinal: South Korea vs. Germany

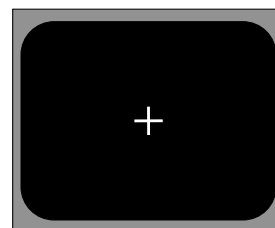




## Experimental Task



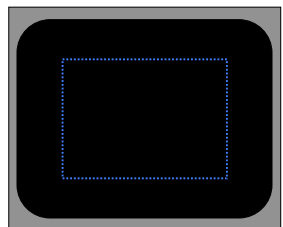
## Experimental Task



Start of trial:  
display of fixation  
cross (1.5 s)



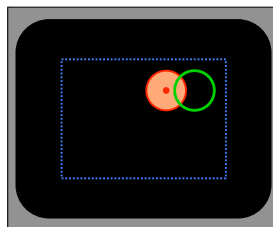
### Experimental Task



Display of response area,  
500 ms before  
target onset  
(114.2 mm x 80.6 mm)



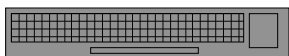
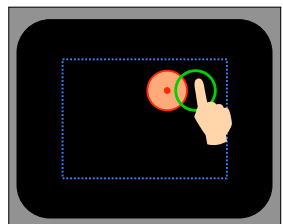
### Experimental Task



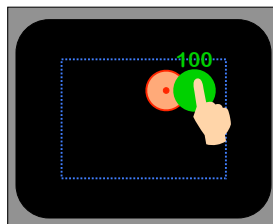
Target display (700 ms)



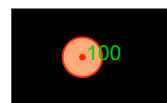
### Experimental Task



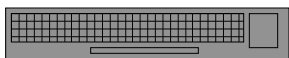
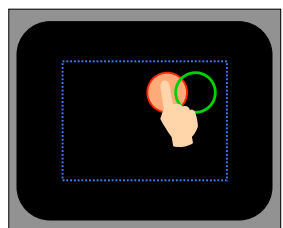
### Experimental Task



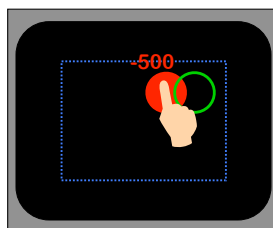
The green target is hit:  
+100 points



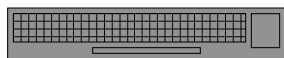
### Experimental Task



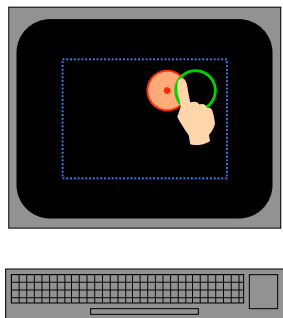
### Experimental Task



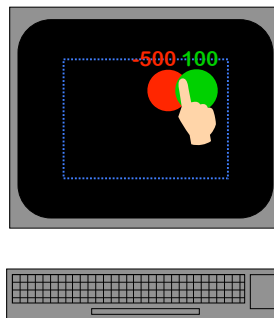
The red target is hit:  
-500 points



### Experimental Task



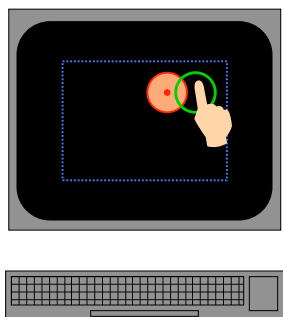
### Experimental Task



Scores add if both targets are hit:



### Experimental Task

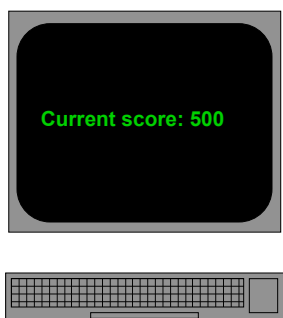


### Experimental Task



The screen is hit later than 700 ms after target display: -700 points.

### Experimental Task

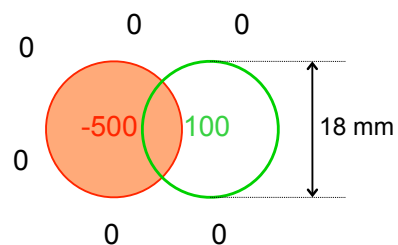


End of trial

### Experimental Task

Rapidly touch a point with your fingertip.

Responding after the time limit: -700 points

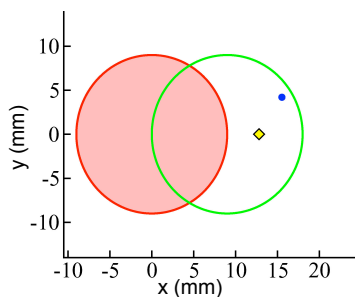


*What should you do?*



### Thought Experiment

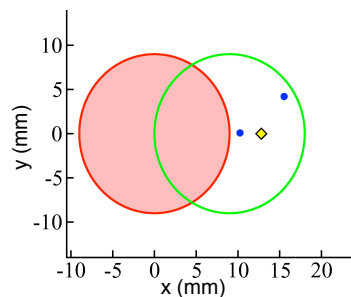
● : -500      ○ : 100 points (2.5  $\phi$ )



$\sigma = 4.83$  mm

### Thought Experiment

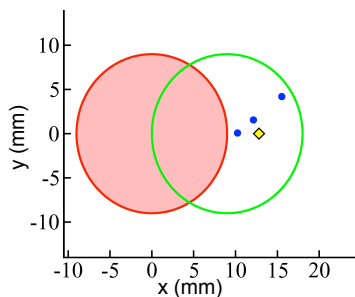
● : -500      ○ : 100 points (2.5  $\phi$ )



$\sigma = 4.83$  mm

### Thought Experiment

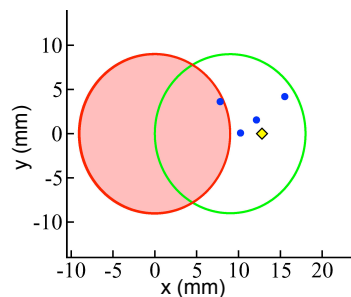
● : -500      ○ : 100 points (2.5  $\phi$ )



$\sigma = 4.83$  mm

### Thought Experiment

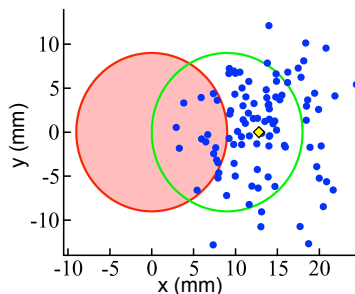
● : -500      ○ : 100 points (2.5  $\phi$ )



$\sigma = 4.83$  mm

### Thought Experiment

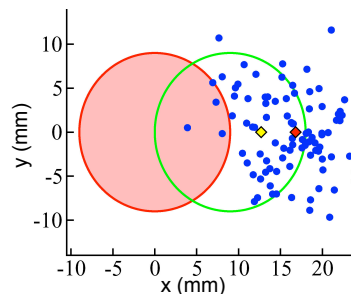
● : -500      ○ : 100 points (2.5  $\phi$ )



$\sigma = 4.83$  mm

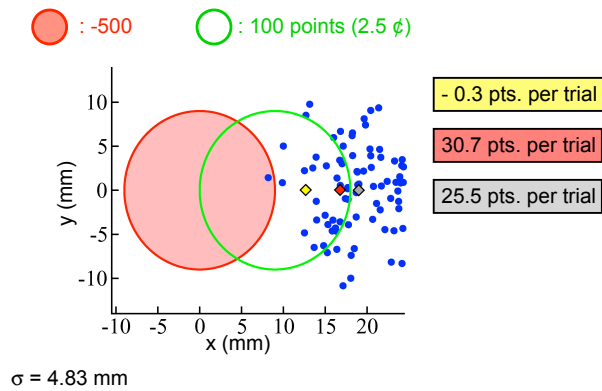
### Thought Experiment

● : -500      ○ : 100 points (2.5  $\phi$ )

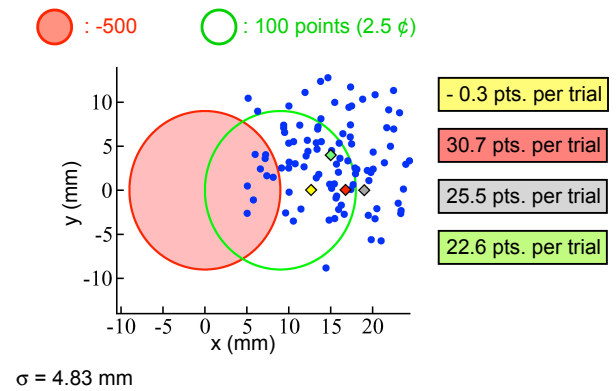


$\sigma = 4.83$  mm

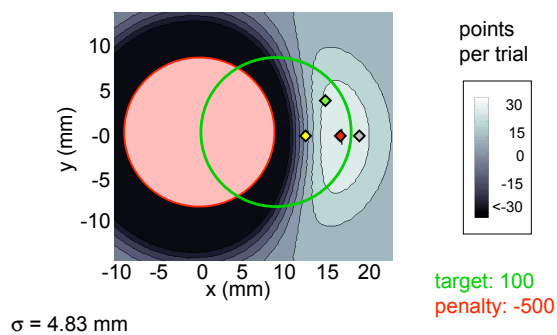
## Thought Experiment



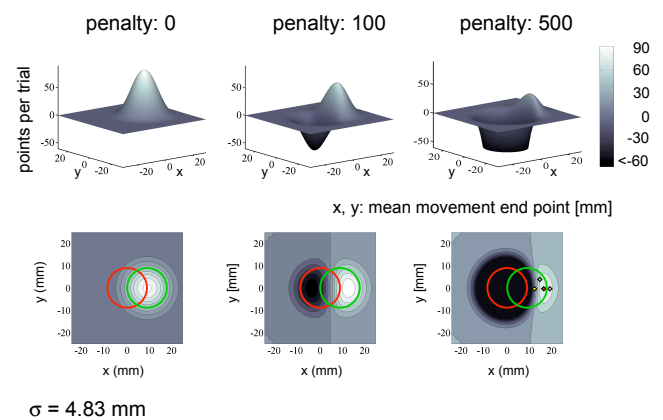
## Thought Experiment



Expected gain as function of mean movement end point (x,y):

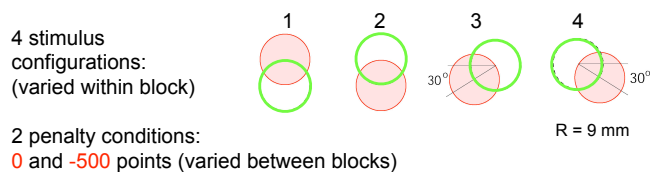


## Thought Experiment



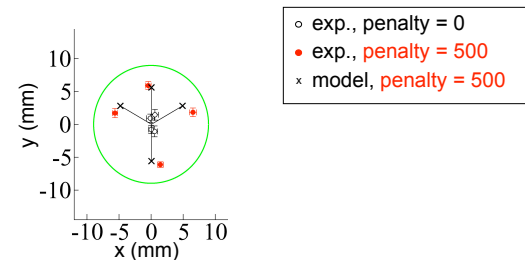
## Experiment: Movement Under Risk

Movement endpoints in response to novel stimulus configurations.



## Results

Comparison with experiment



Subject S5,  $\sigma = 2.99$  mm

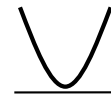
## Summary: Movement Under Risk

Subjects' movement endpoints match those that, for their motor variability and the experimenter-imposed task conditions and risk, *do* maximize expected gain.

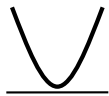
Subjects appear to do this, even when confronting novel configurations, from the first trial, with no apparent learning.

Subjects effectively take into account their own motor variability in planning movements.

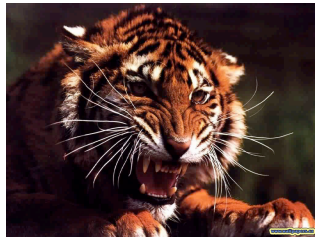
Trommershäuser, Maloney & Landy (2003). *JOSA A*, 20,1419.



*loss function?*



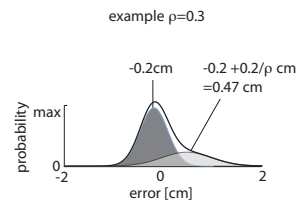
*loss function?*



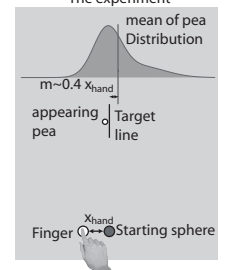
*loss function!*

## Estimating the human cost function: Körding & Wolpert (2004)

A Constructing the distribution

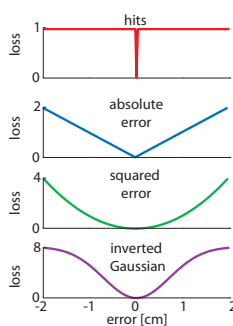


B The experiment

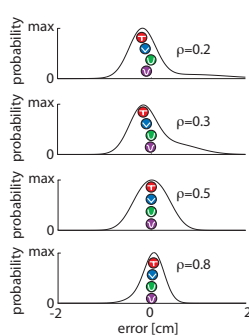


## Estimating the human cost function: Körding & Wolpert (2004)

C Possible loss functions



D Distributions and optimal means



## Estimating the human cost function: Körding & Wolpert (2004)

