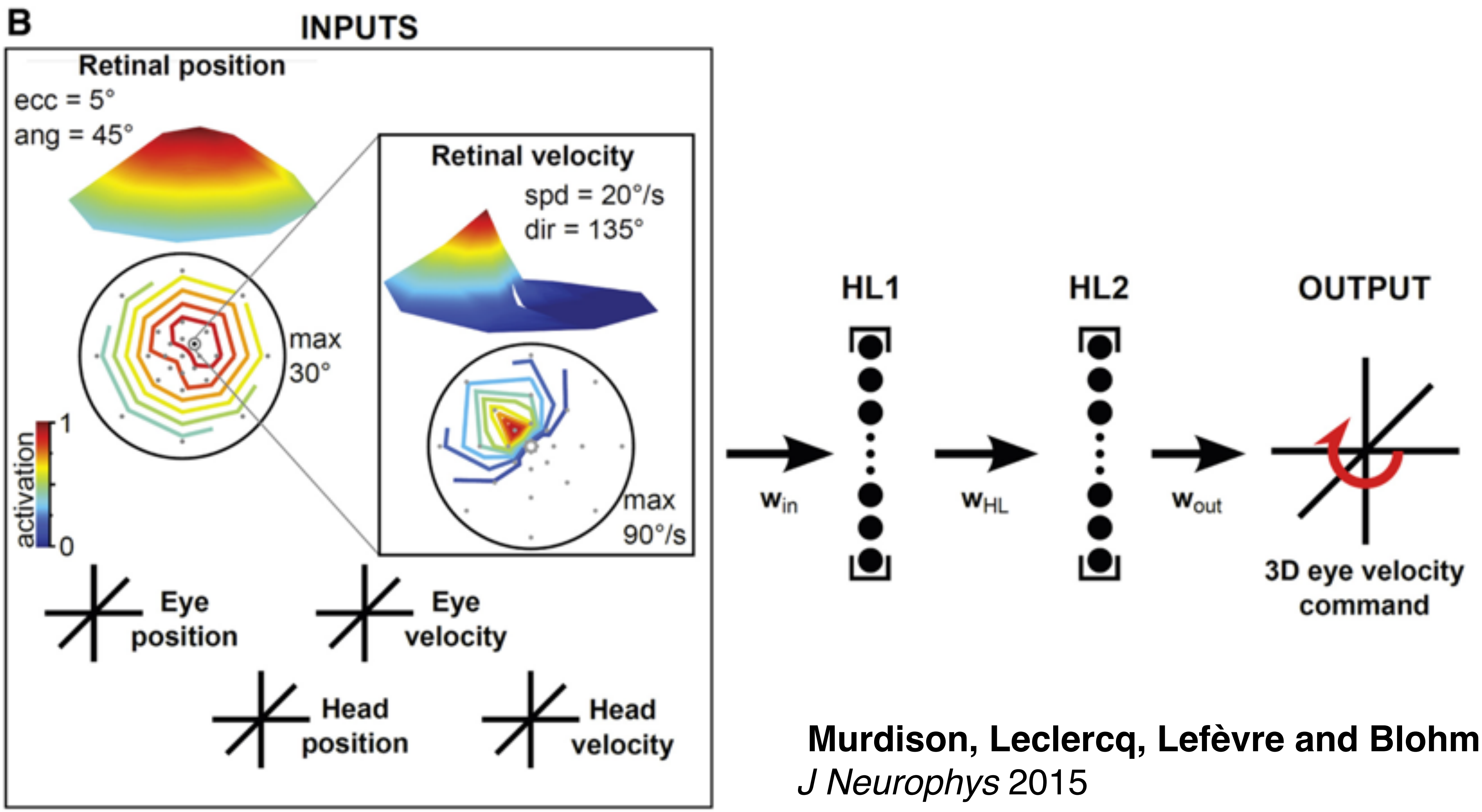
Rate-based artificial neural networks and error backpropagation learning

Scott Murdison
Machine learning journal club
May 16, 2016



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J Neurophys 2015

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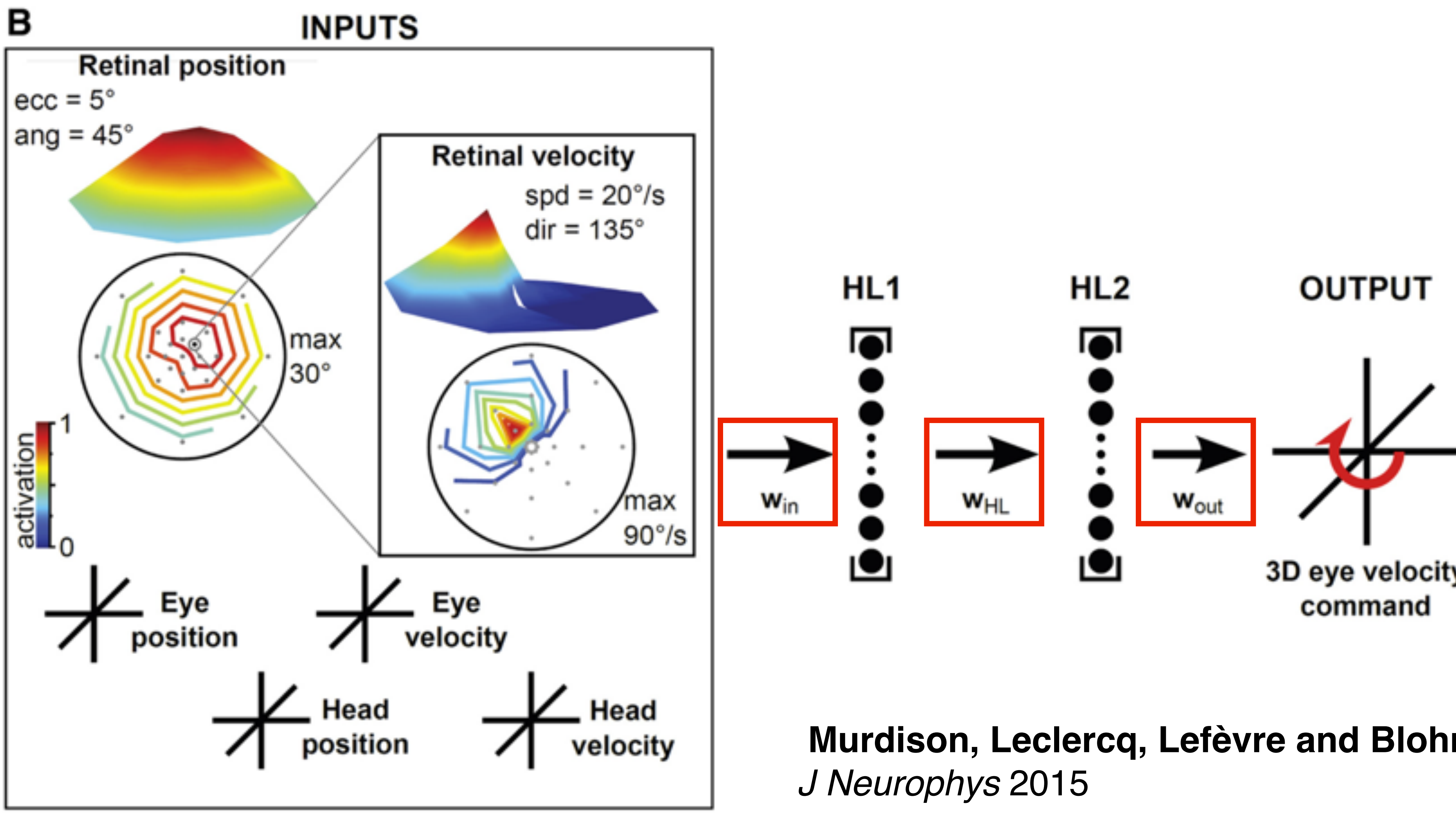

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- stock market predictions
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Geoff Hinton sweet intro vid

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> In the Fall of 1992, for a class project in Artificial Intelligence, I designed a neural network to locate facial features in images. The one hundred images I used came from the underclassmen section of the 1987 University High School yearbook. They were scanned in at 96 by 128 resolution. I set four of the images aside to comprise the testing set, and for the remaining ninety-six I manually specified the coordinates of the left eye, right eye, nose, and mouth.

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Log-polar maps of left eye, nose, and mouth

polar-transformed images around each feature

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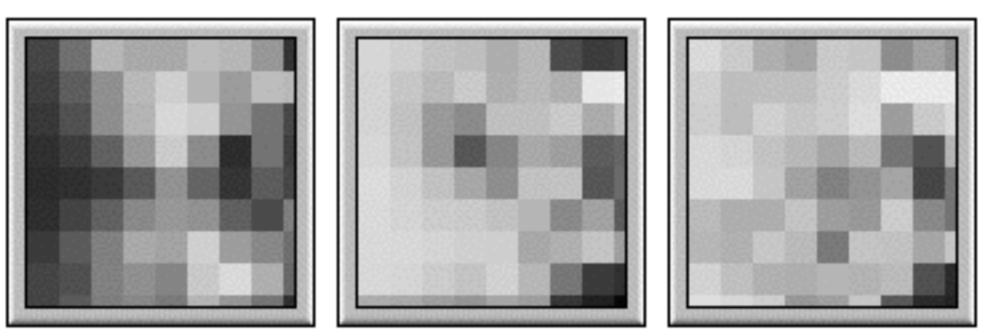

8 by 8 subsamples of the above maps

subsampled images to feed to neural network **…Why?**

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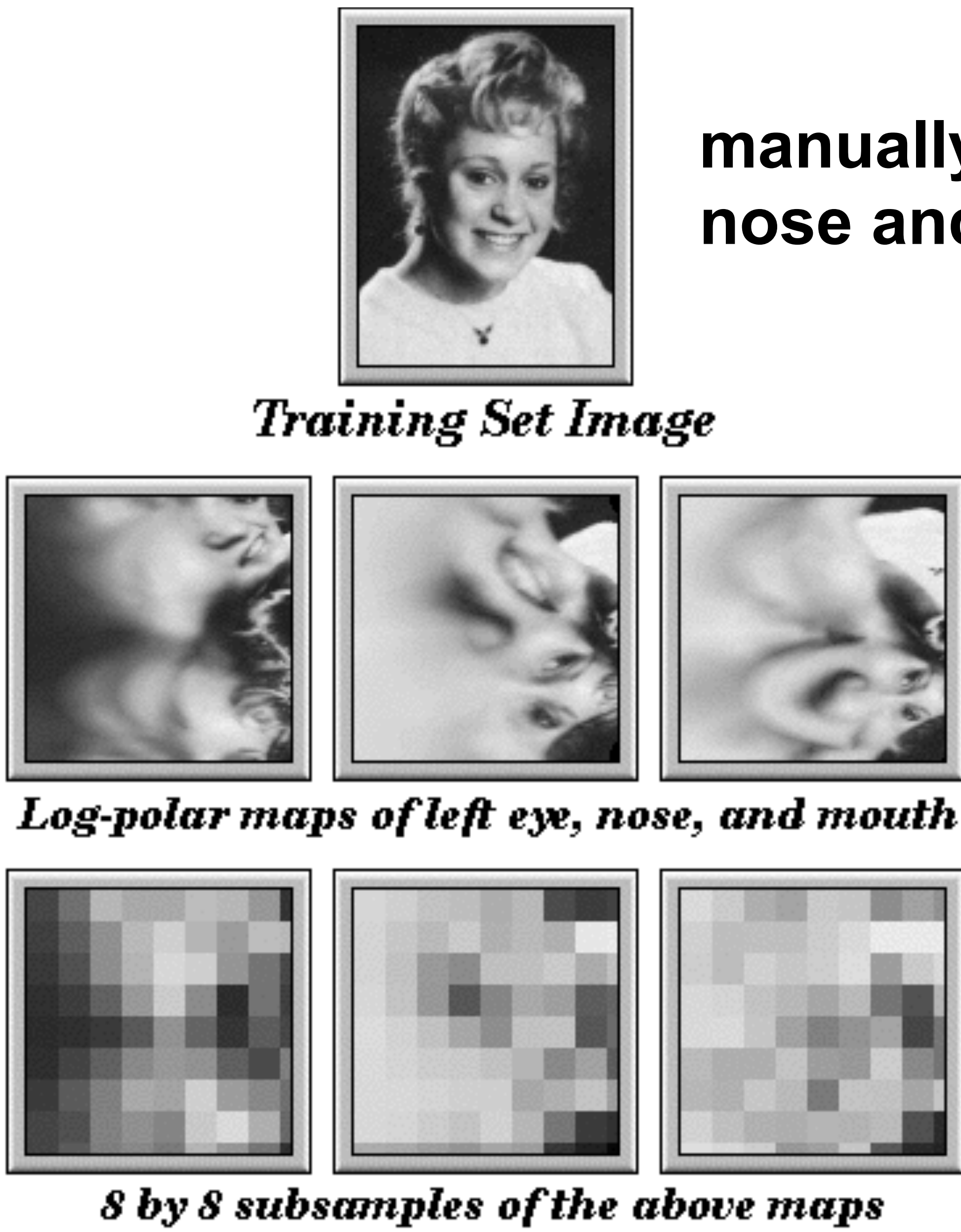

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B. 4**M**B RAM in 1992 = $150 USD

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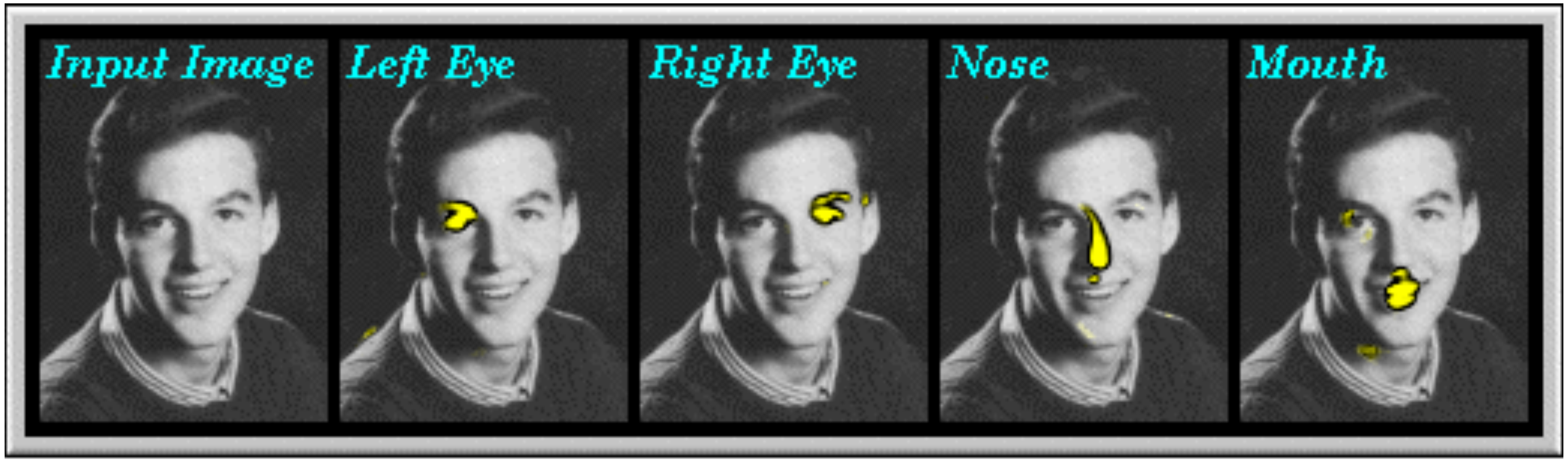

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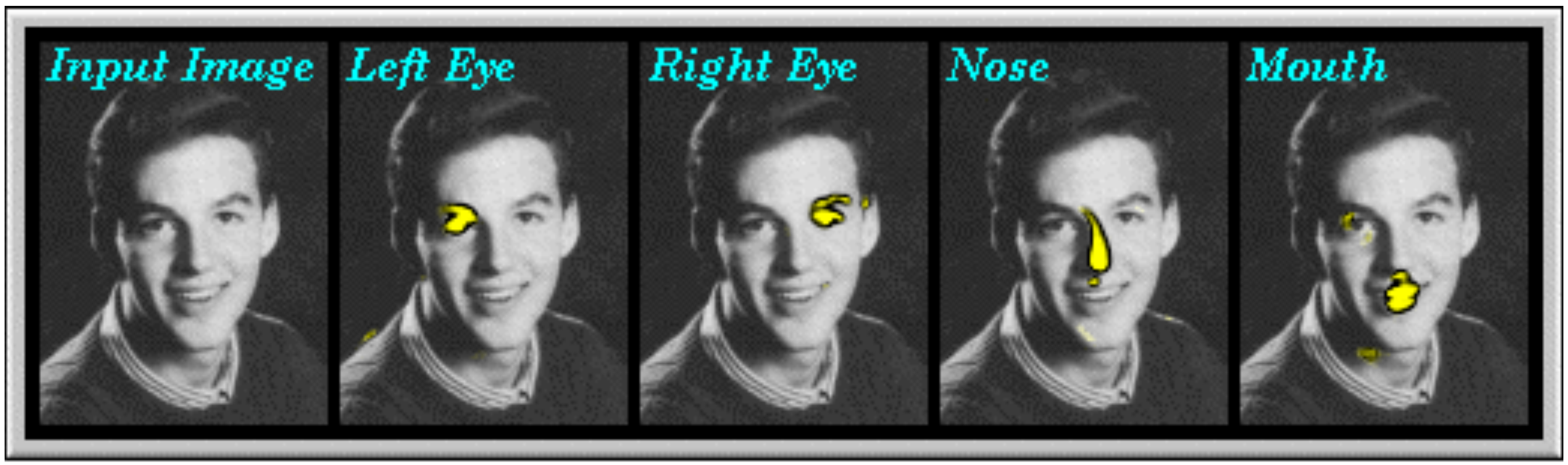

Neural network outputs for a previously unseen face

After training a simple, feedforward network with backprop using yearbook photos it could successfully detect each eye, nose and mouth in previously unseen photo!

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time-resolved
(spiking dynamics, variability)
several levels of abstraction available
(H-H, leaky integrate-and-fire, Izhikevich)

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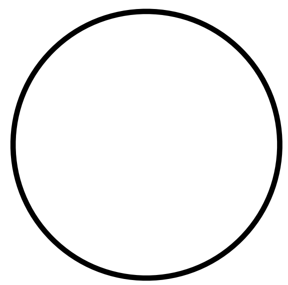
General architecture

Simplest network is the perceptron



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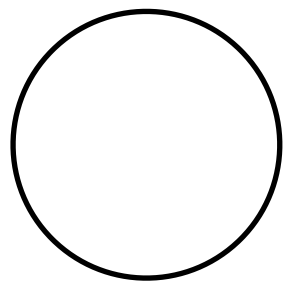
$r_1^{\,in}$



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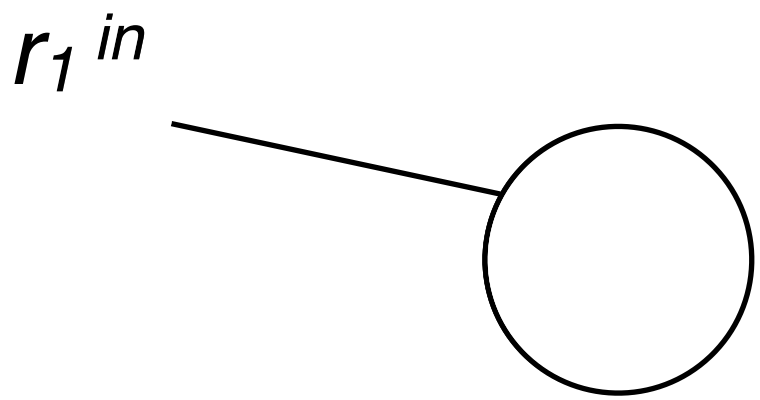

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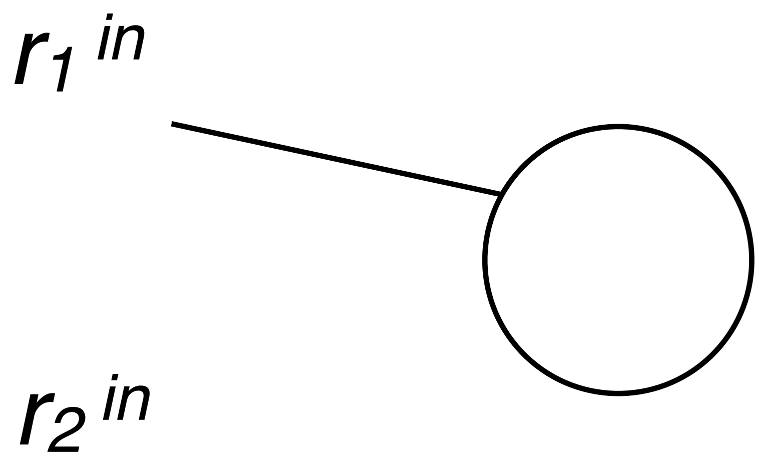

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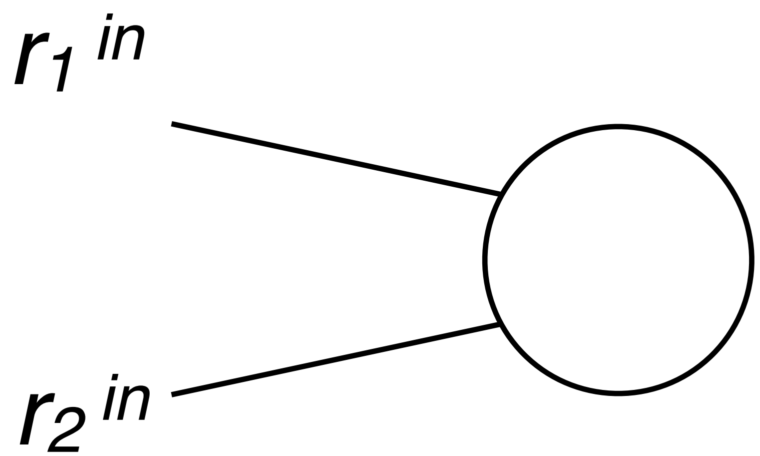

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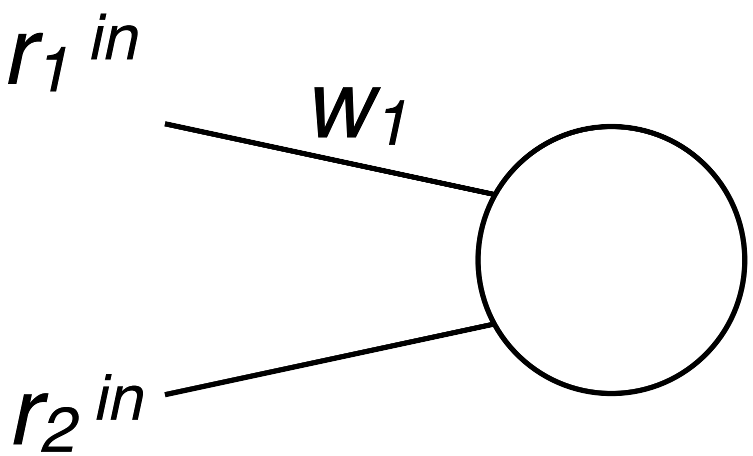

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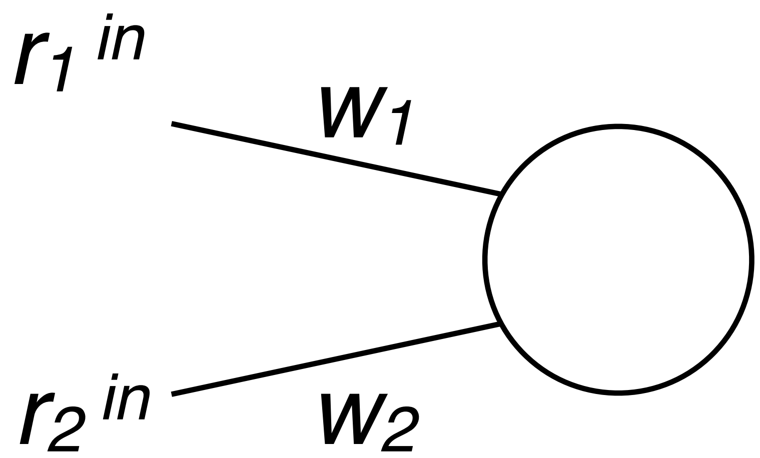

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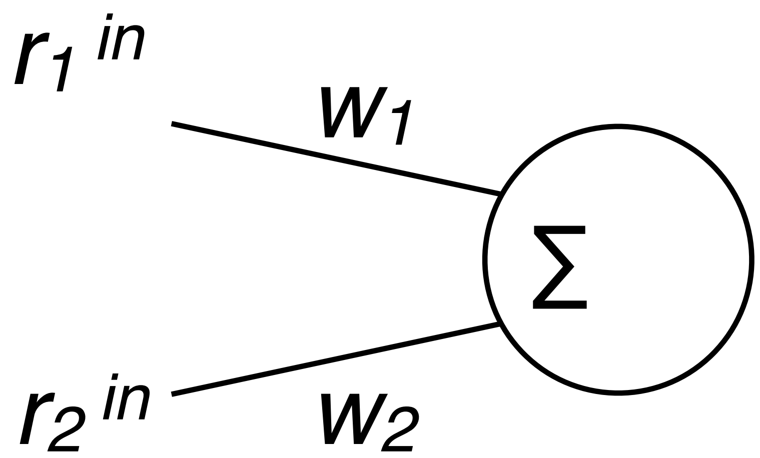

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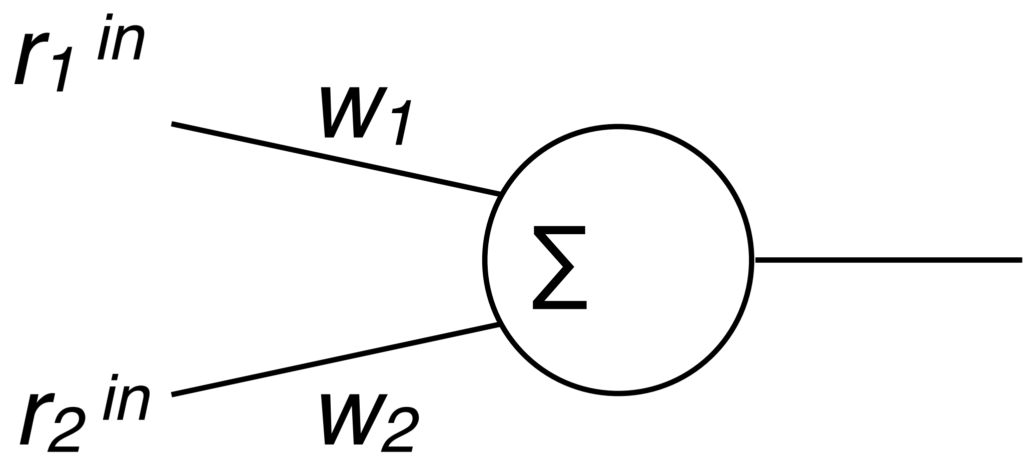

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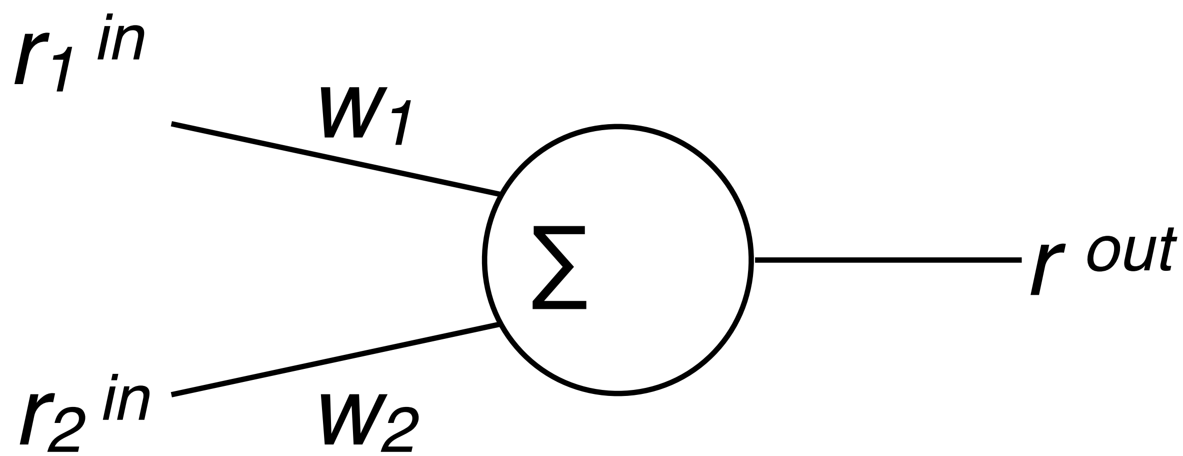

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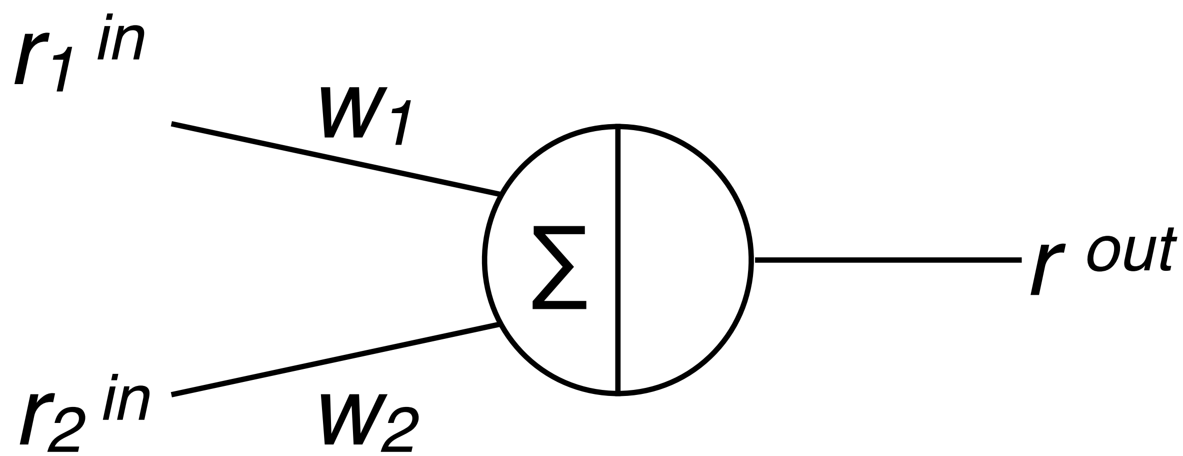

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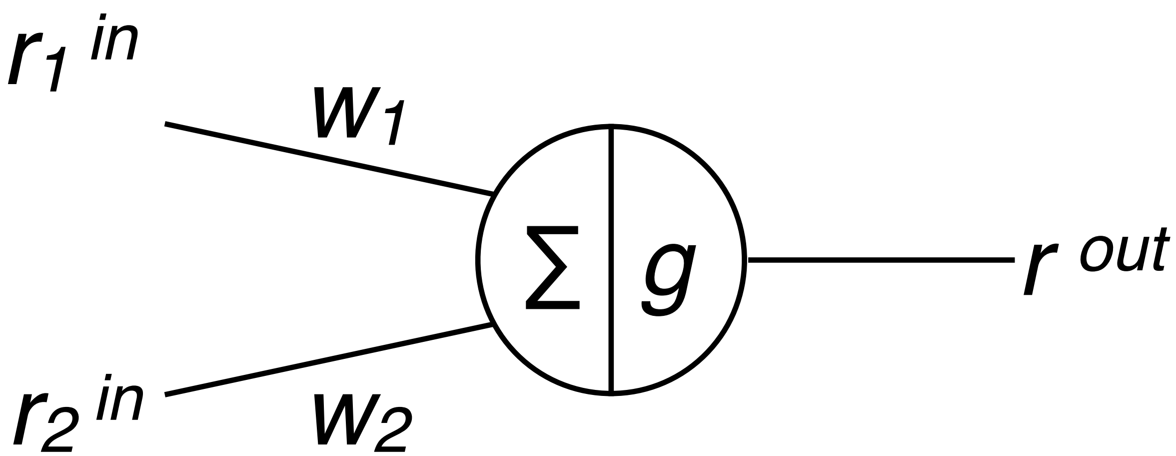

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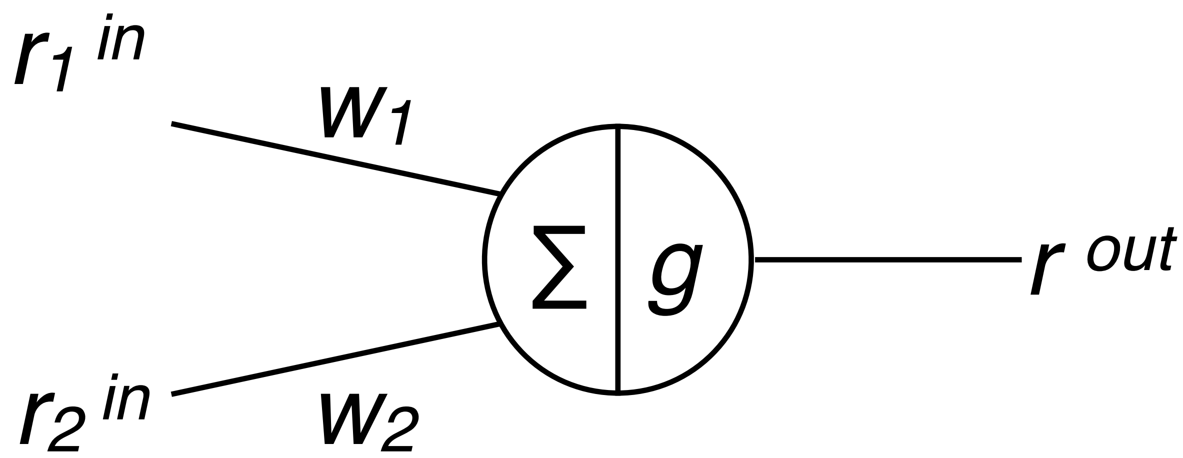

Transfer function *g(∑)*
general engineering term used to describe output of some processing unit as a function of input

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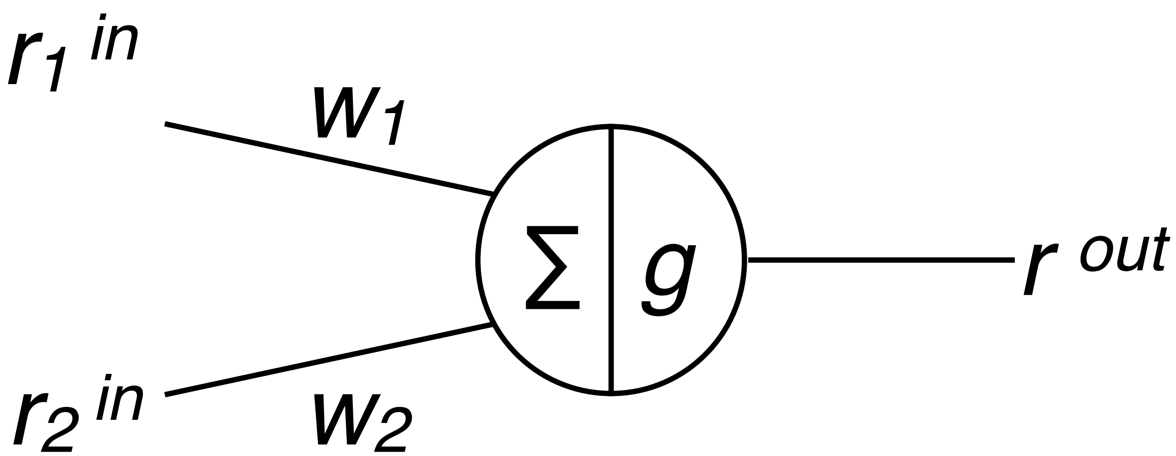

Transfer function $g(\sum)$
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if $r_i^{in} = x_i$ and $r^{out} = y$

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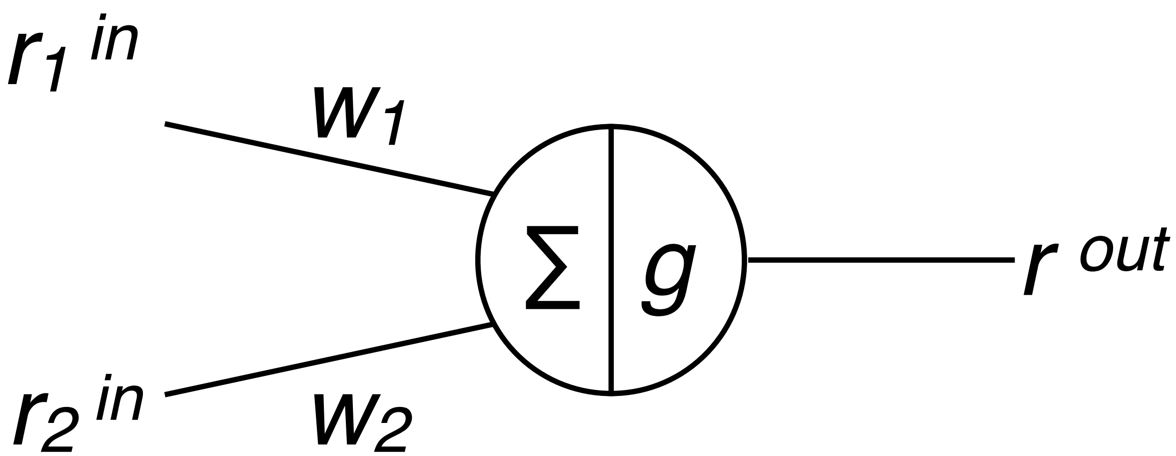

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then $y = g(w_1 \cdot x_1 + w_2 \cdot x_2)$

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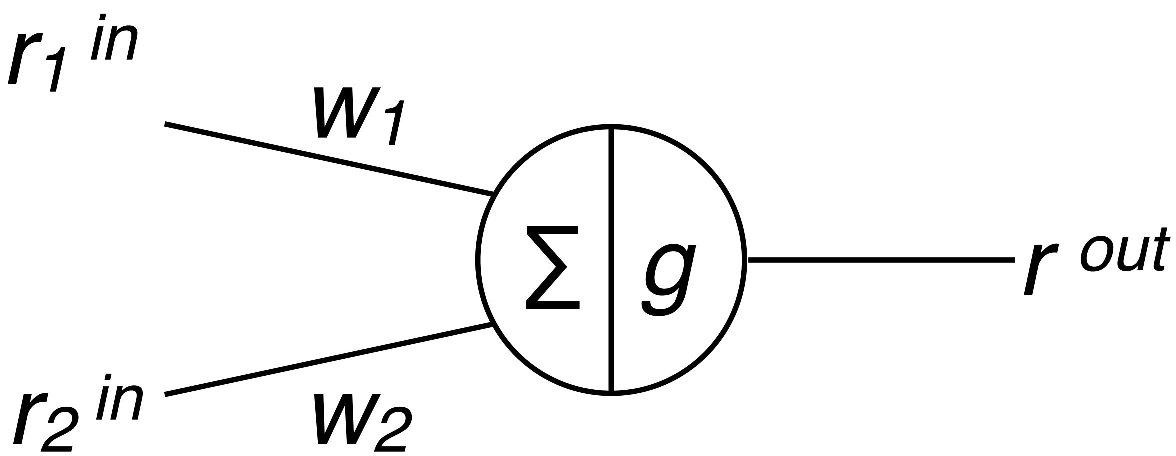

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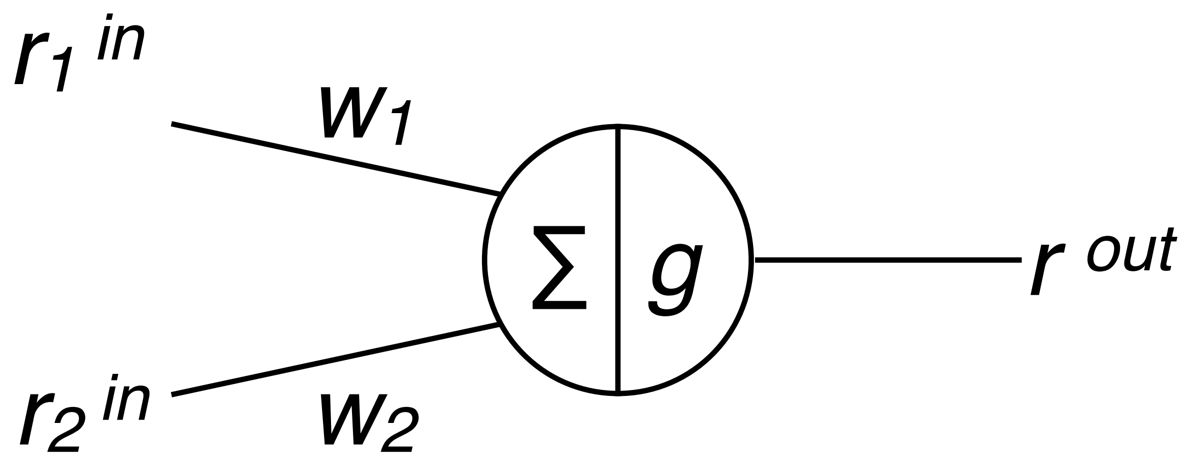

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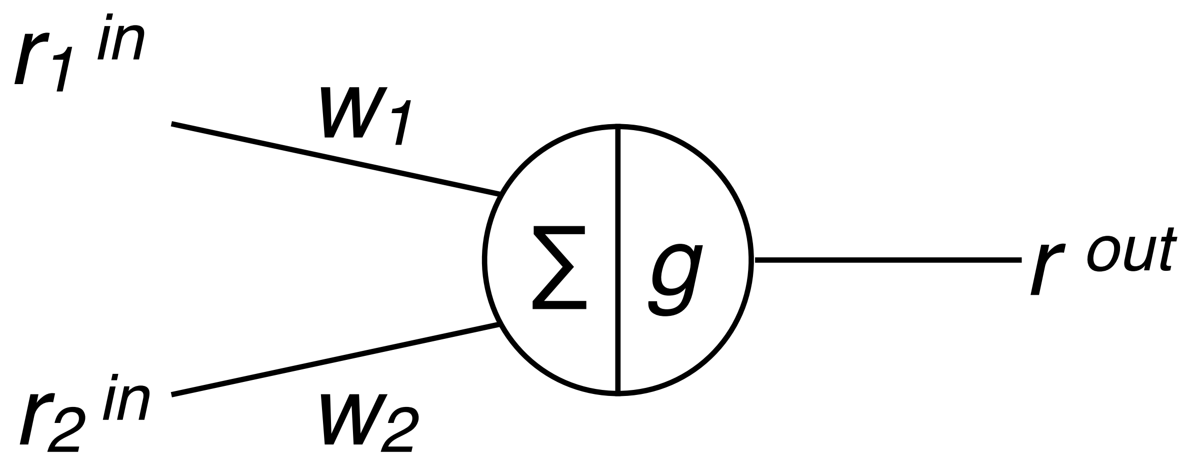

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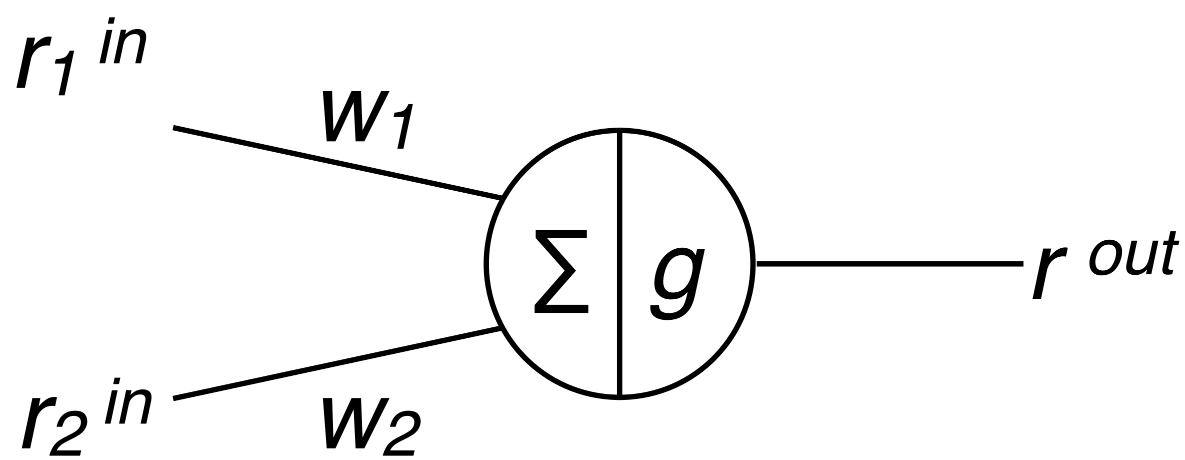
General single-layer mapping

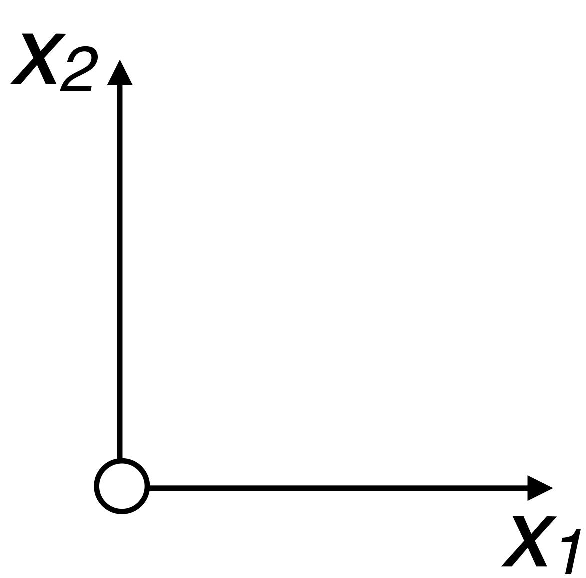
$$r_i^{out} = g(\sum_i w_{ij} r_j^{in}) \Leftrightarrow \mathbf{r}^{out} = g(\mathbf{w}\mathbf{r}^{in})$$

The XOR problem



Boolean OR

x_1	x_2	y
0	0	0



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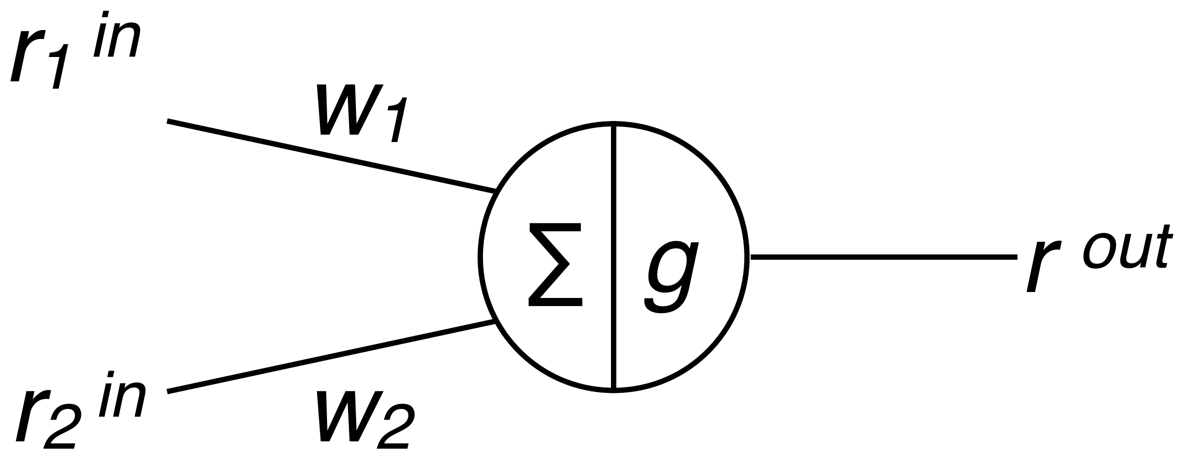

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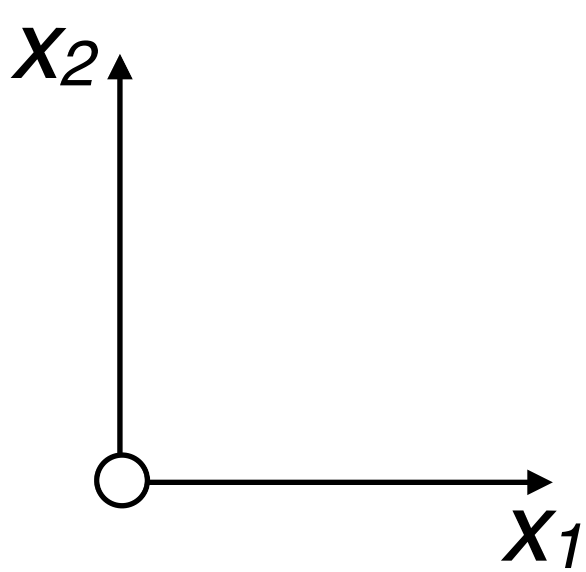

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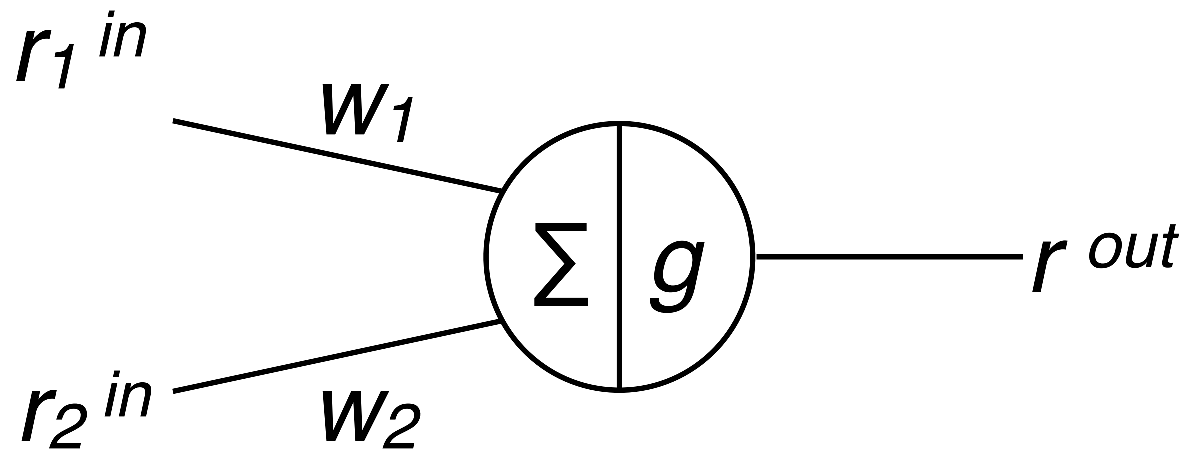

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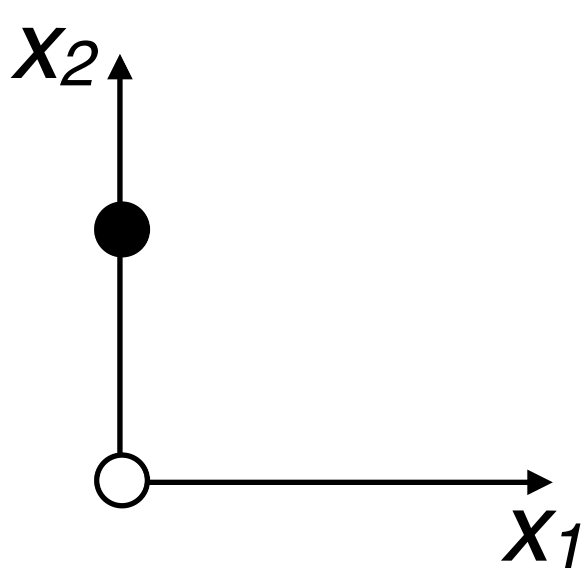

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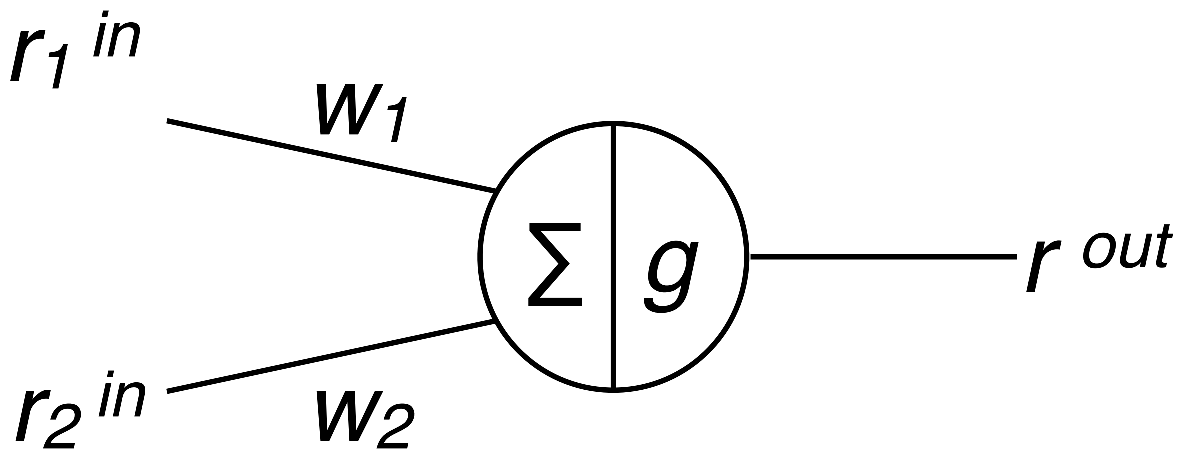

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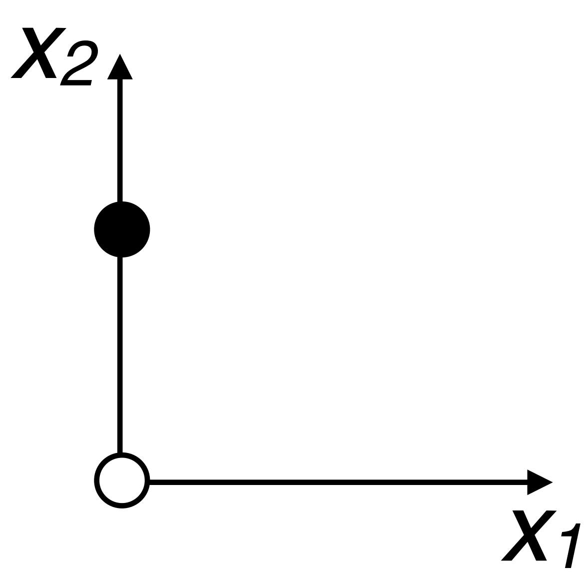

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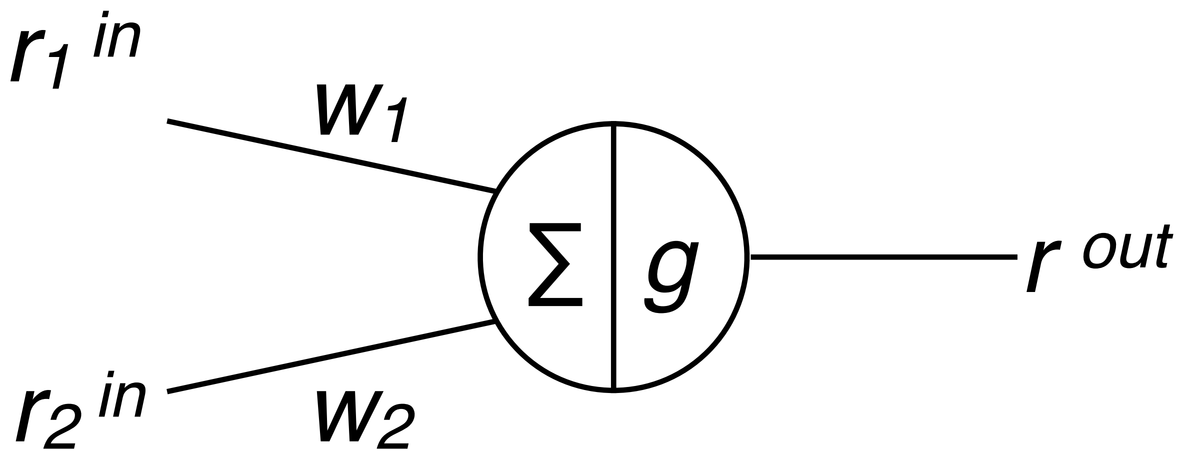

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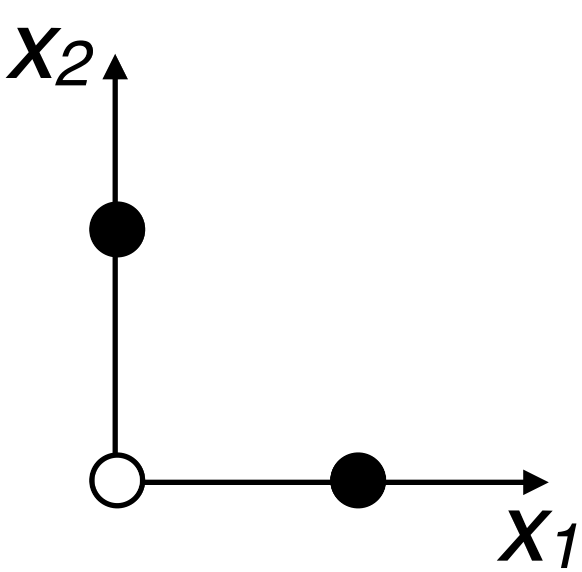

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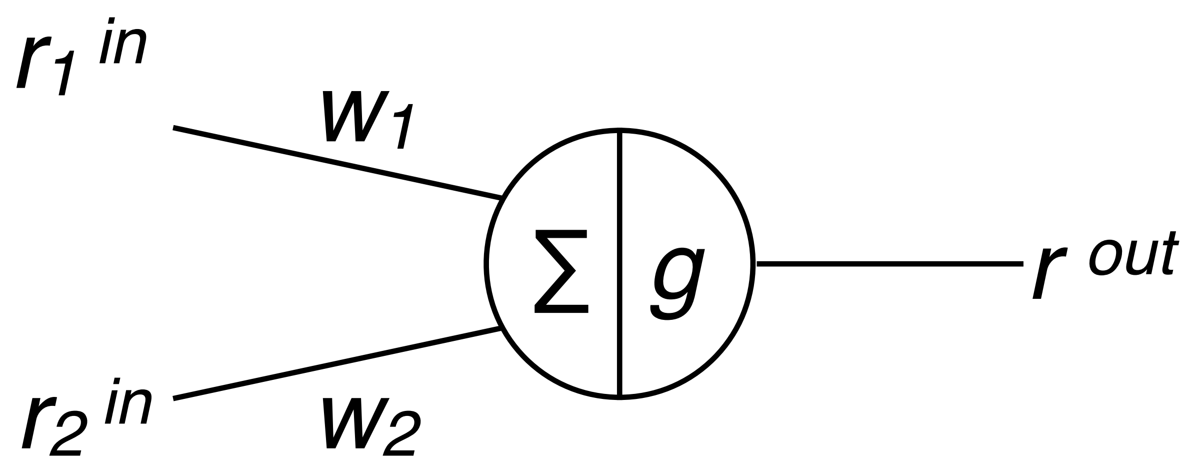

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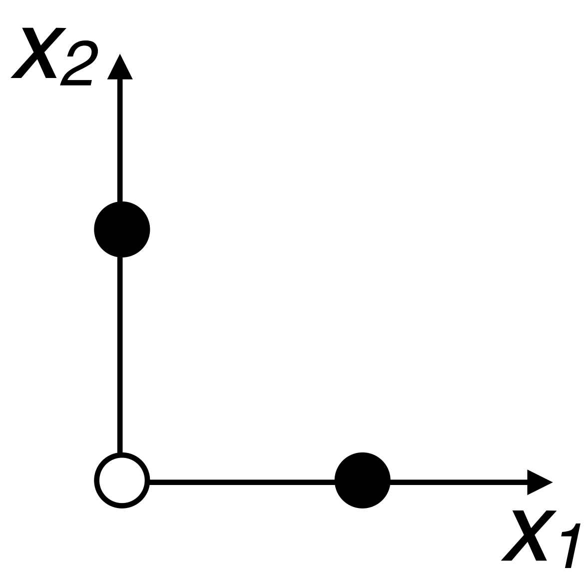

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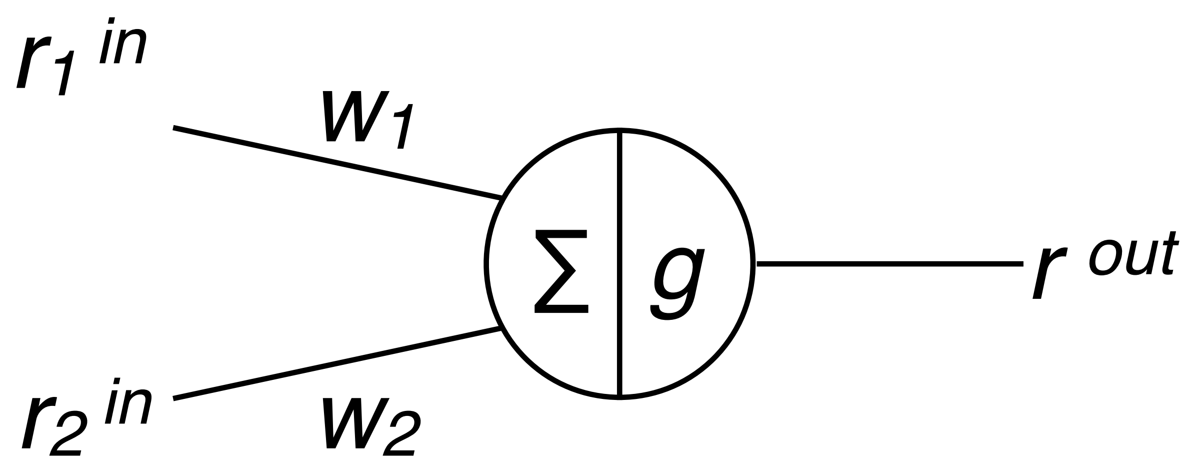

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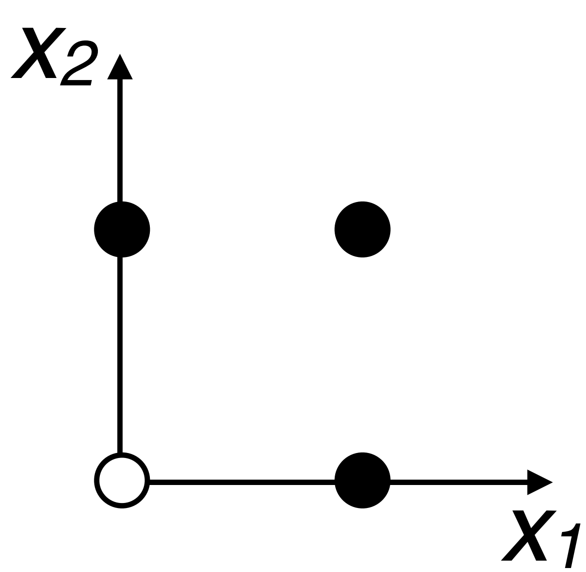

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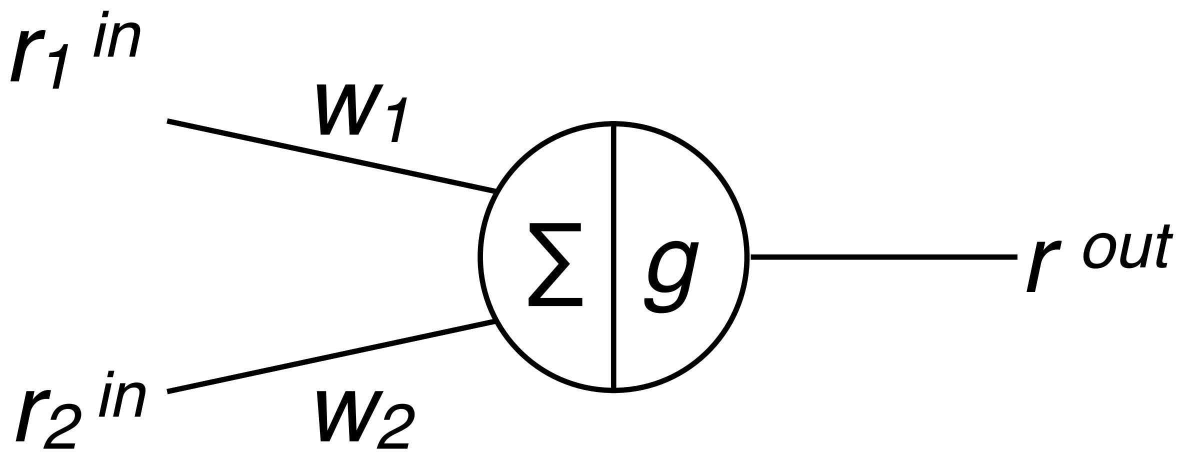

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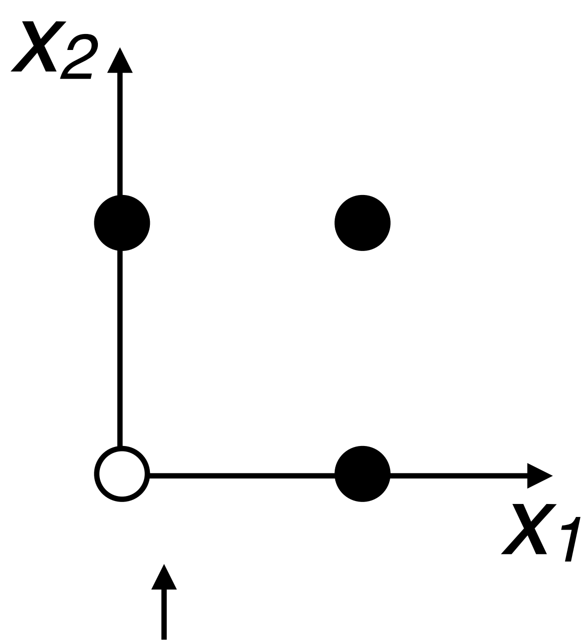

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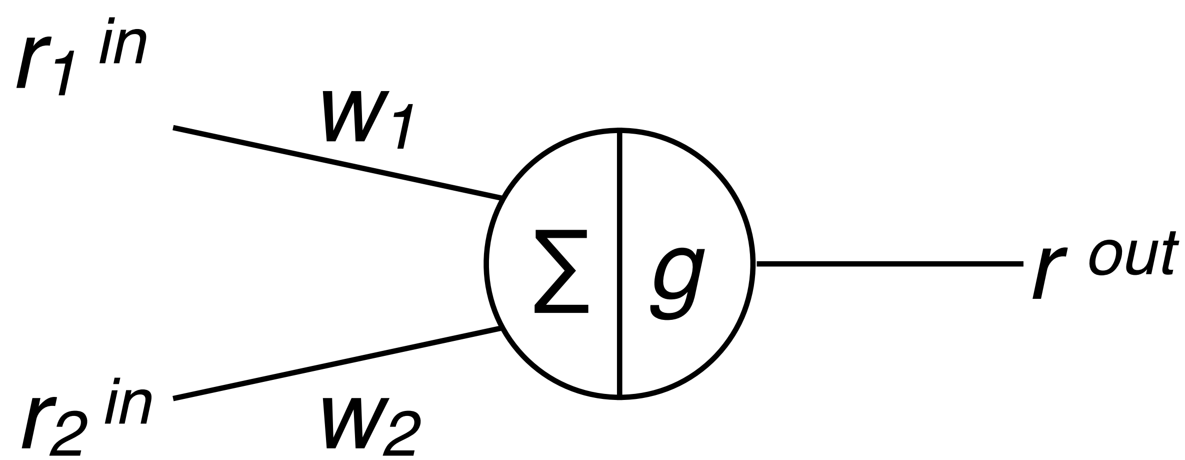

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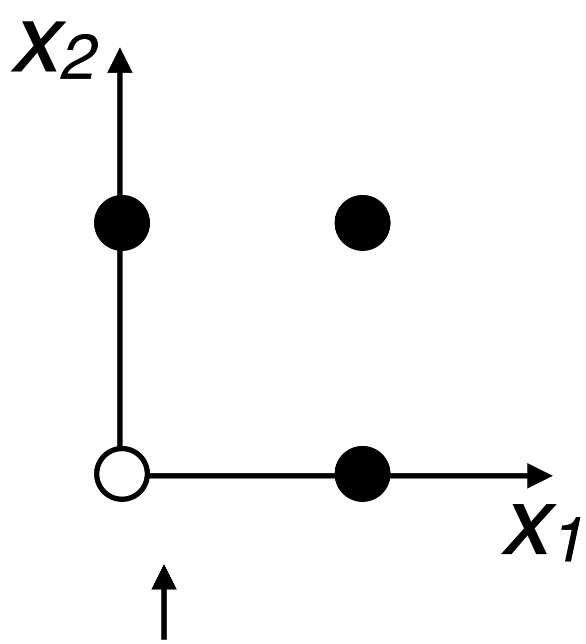

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$$g(x) = \begin{cases} 1 & if\ x > \Theta \\ 0 & else \end{cases}$$

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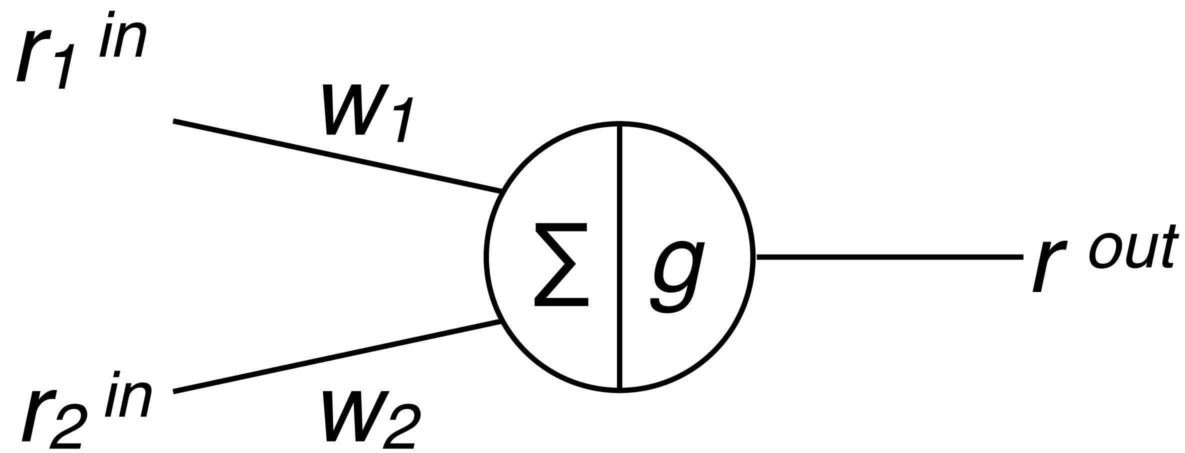

Boolean OR

x_1	x_2	y
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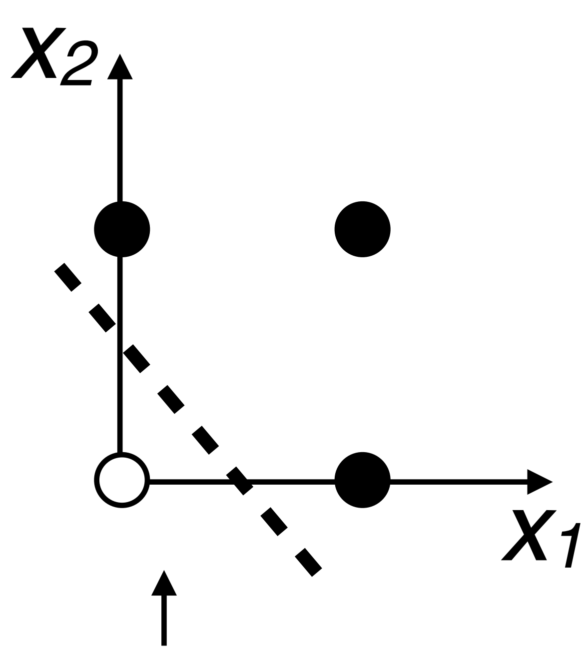
0	1	1
1	0	1
1	1	1



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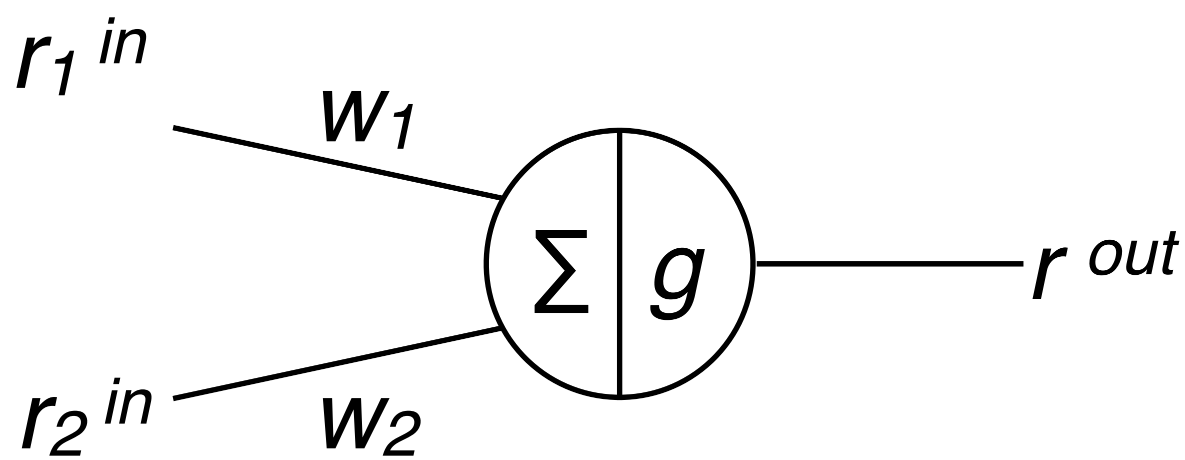

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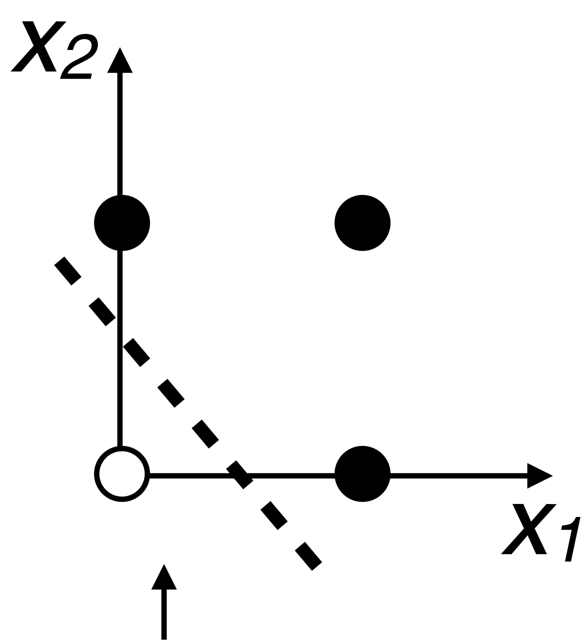

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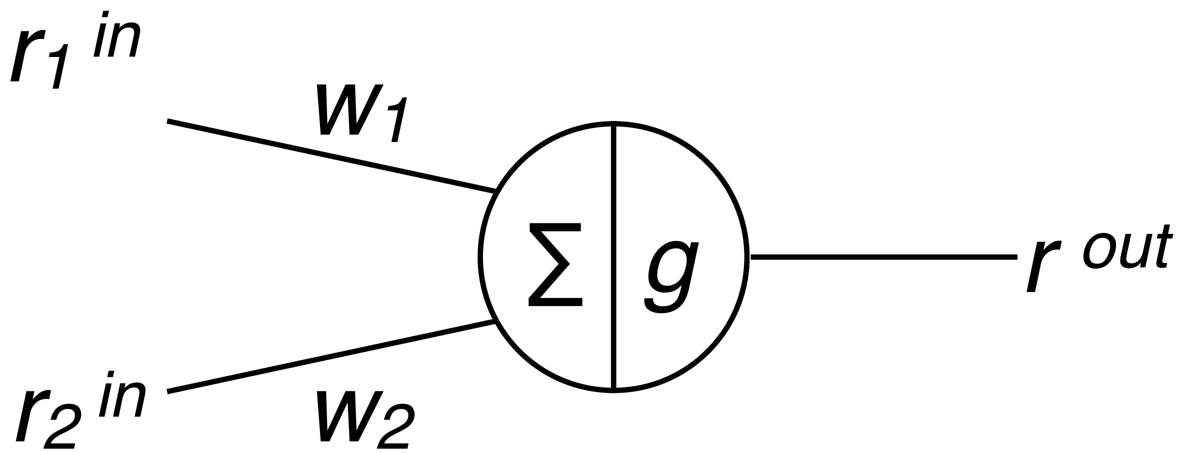

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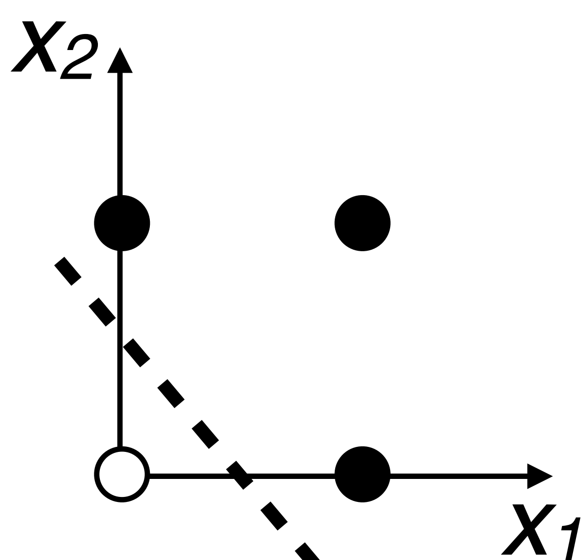

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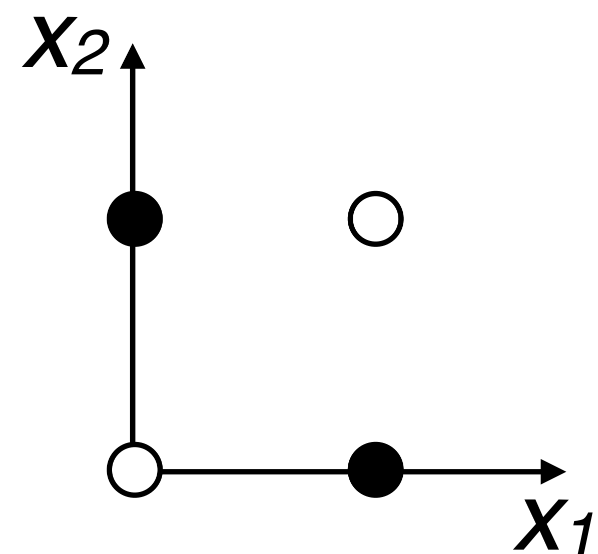

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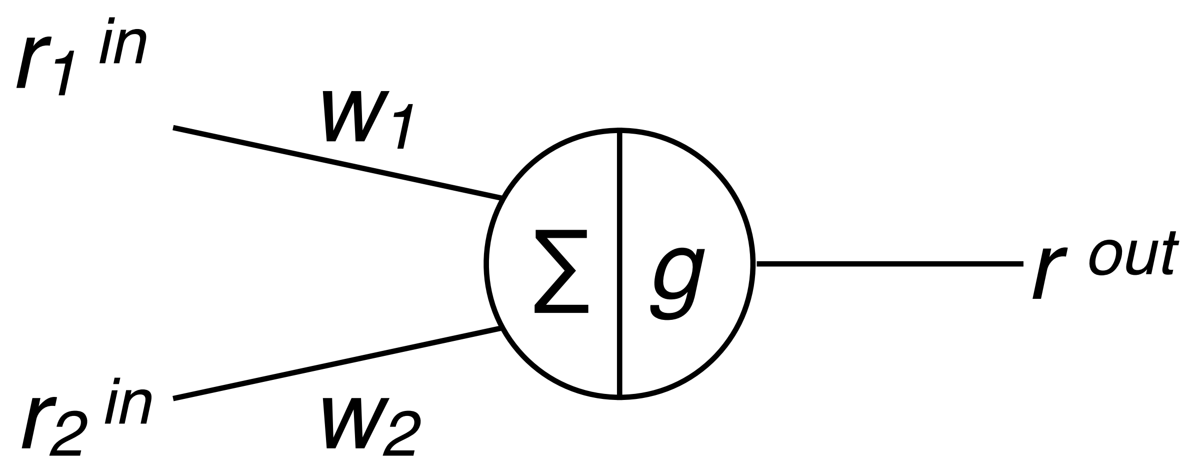

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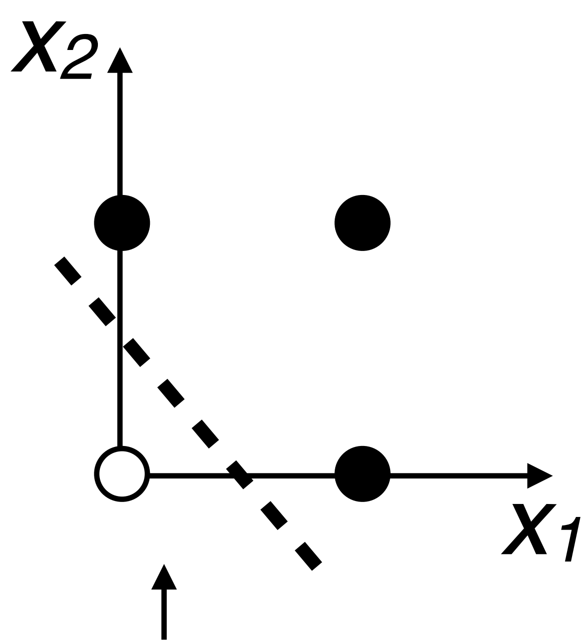

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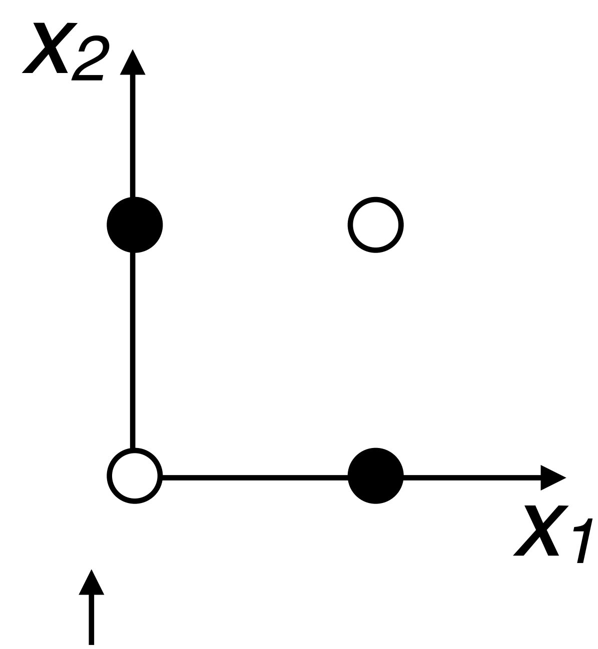
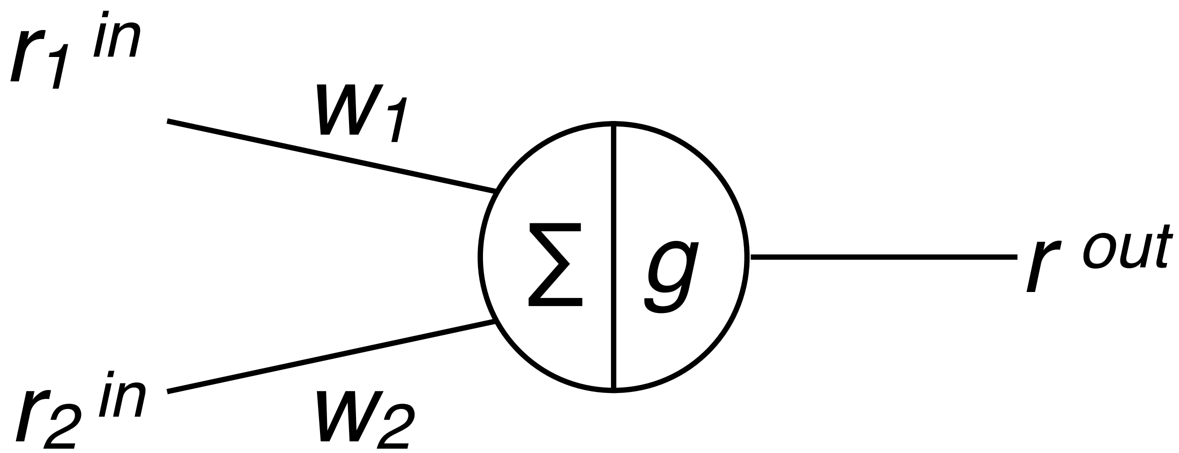


not linearly separable: more complicated!

The XOR problem



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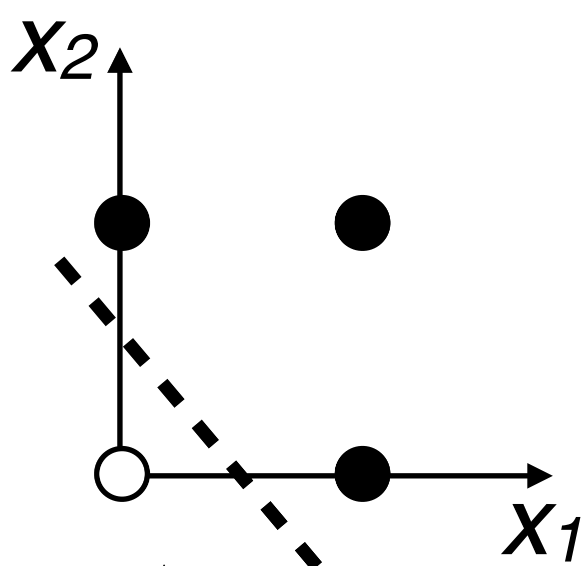

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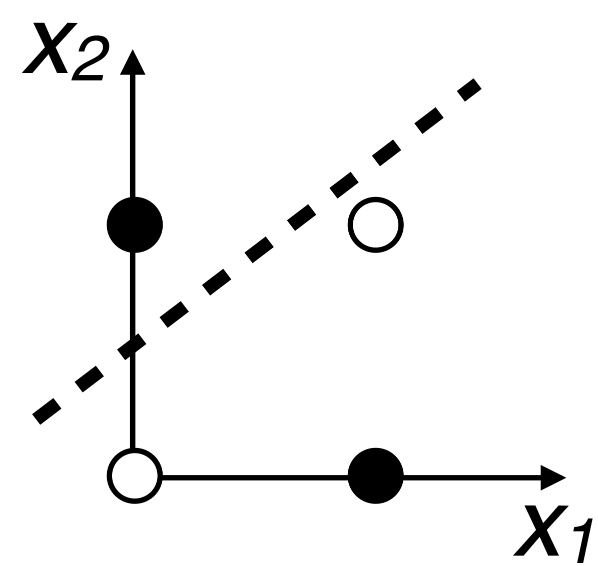

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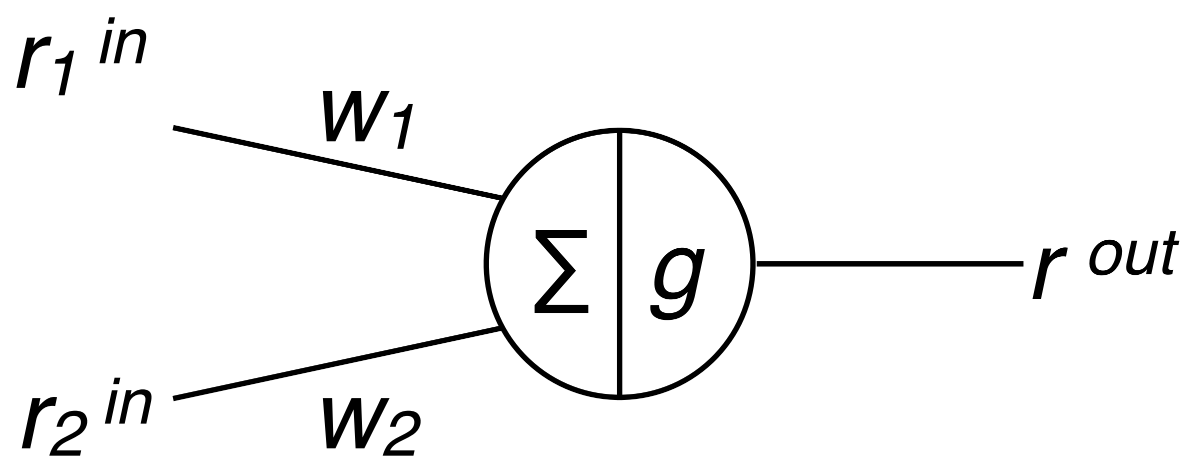

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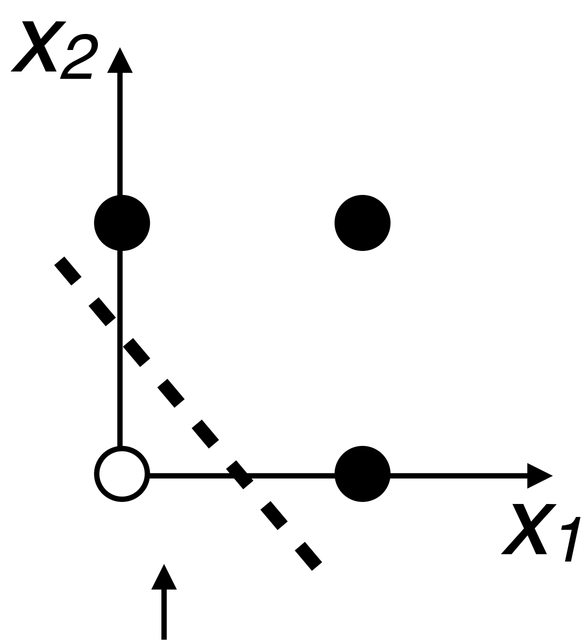

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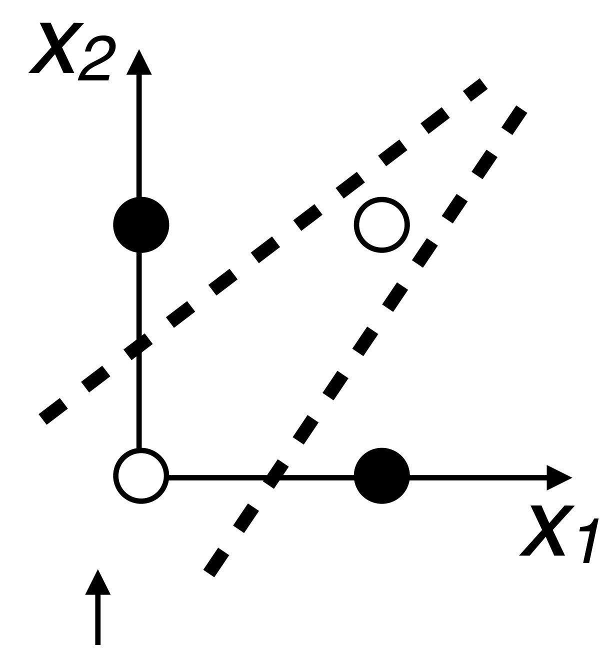

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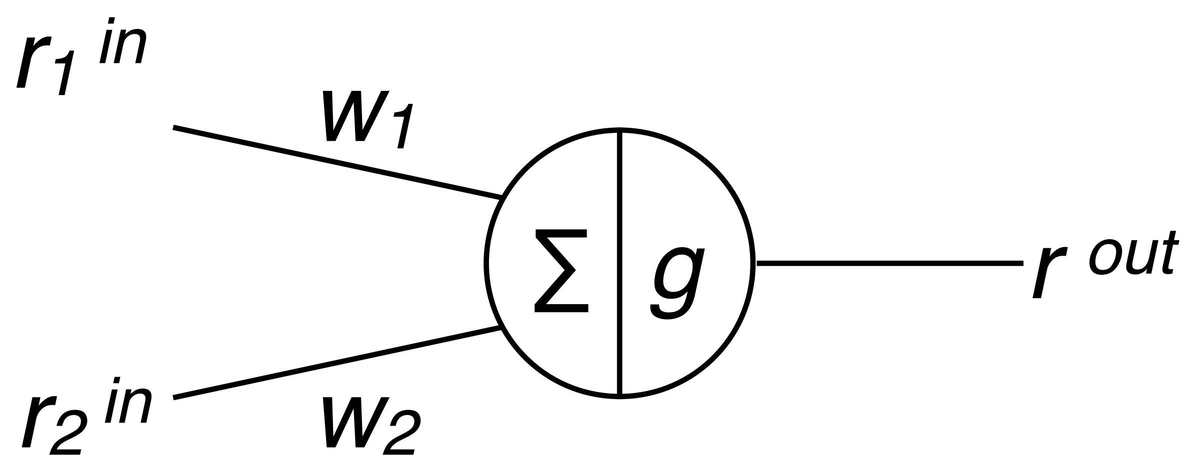

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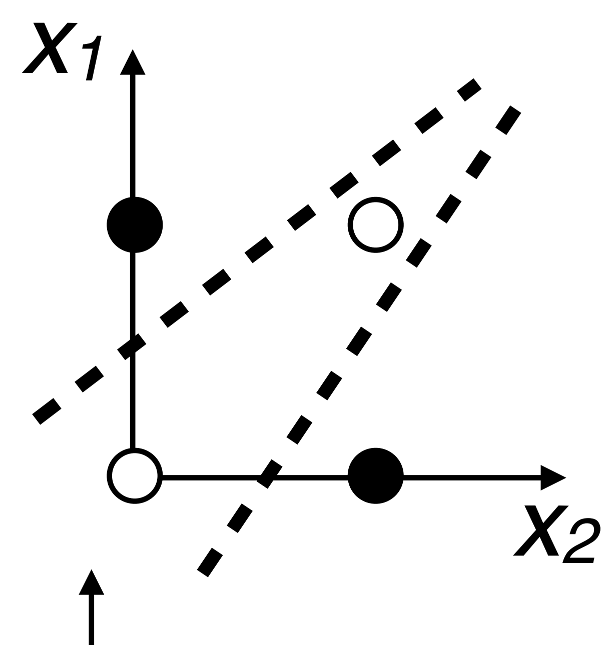

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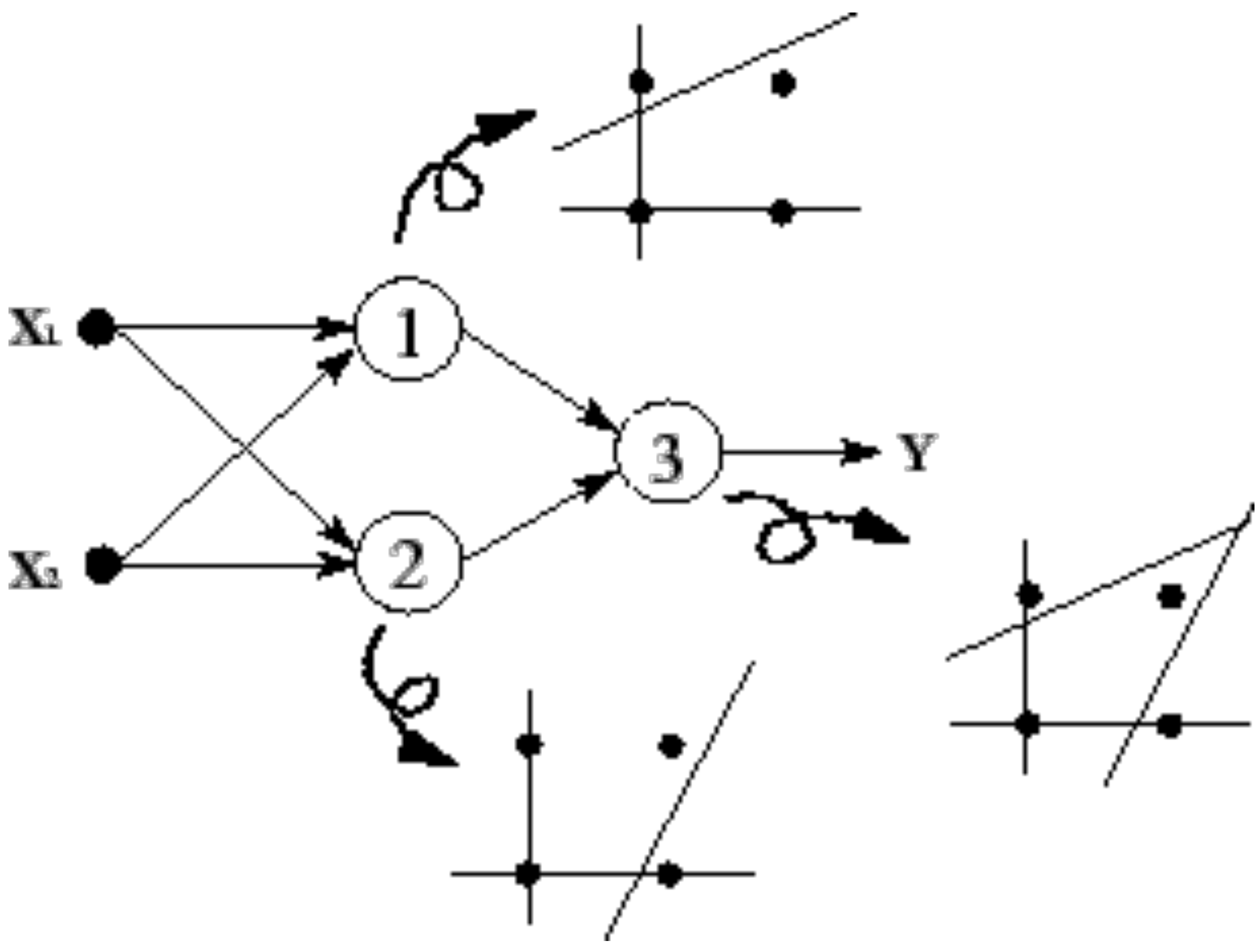
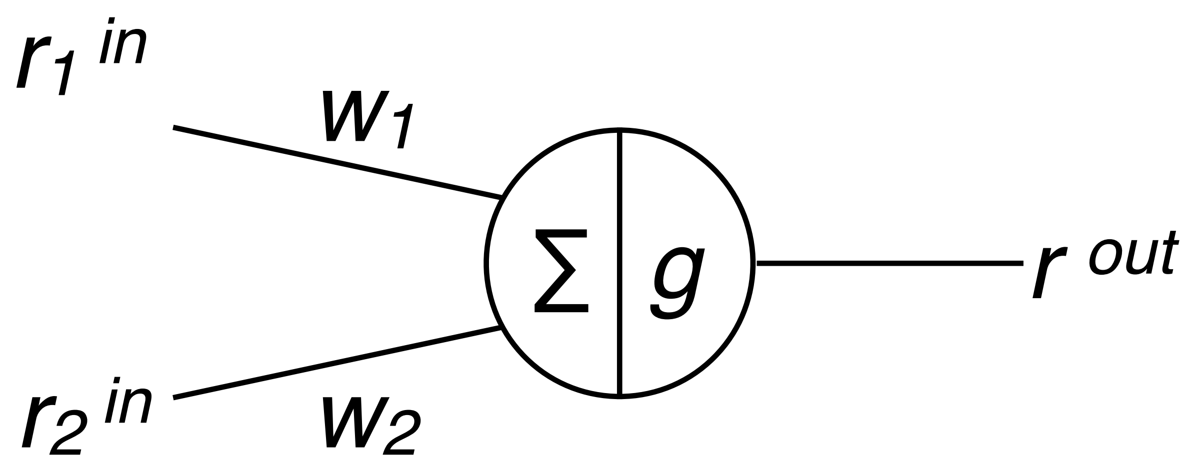
becomes



http://www.ece.utep.edu/research/webfuzzy/docs/kk-thesis/kk-thesis-html/node20.html

The XOR problem



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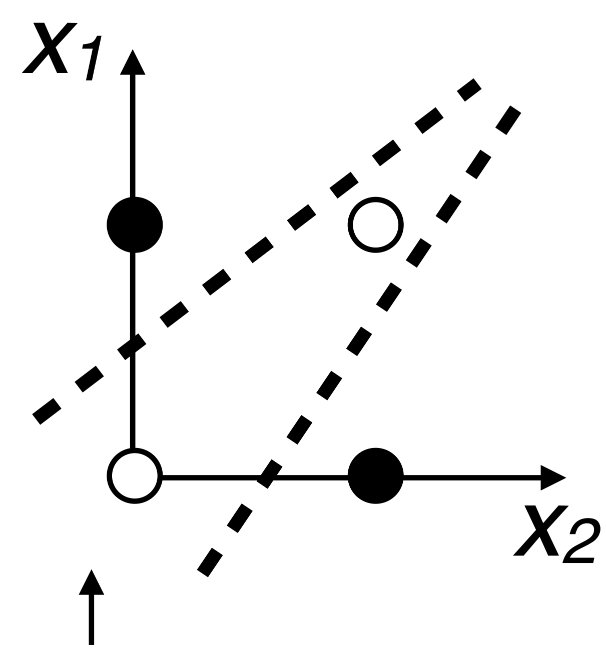

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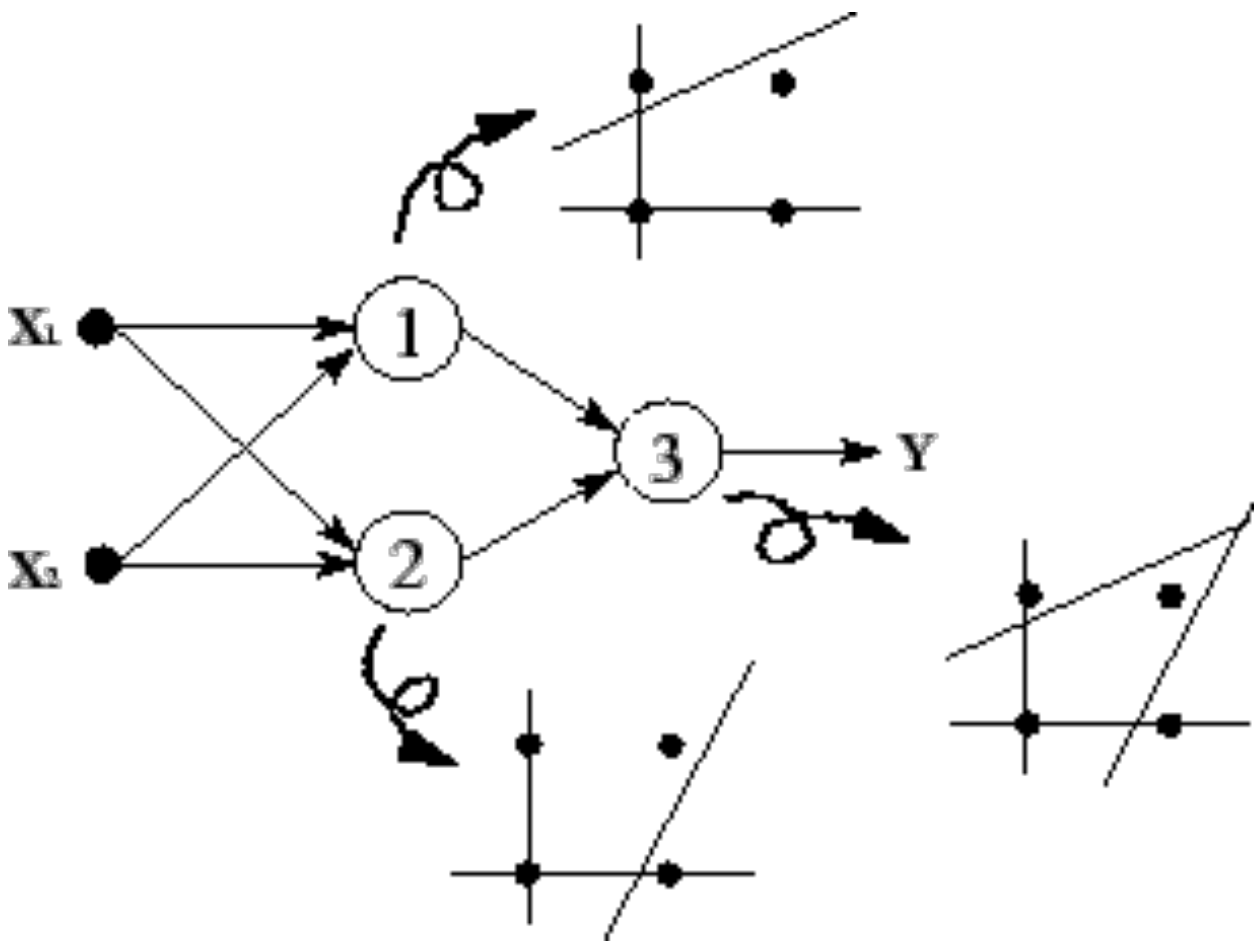

With enough hidden units, multilayer perceptron is the universal function approximator!

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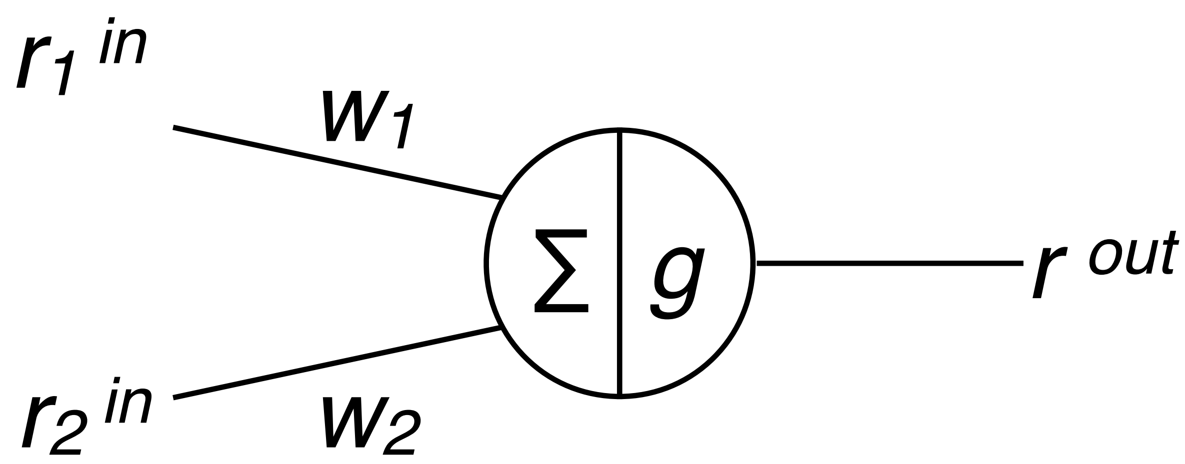

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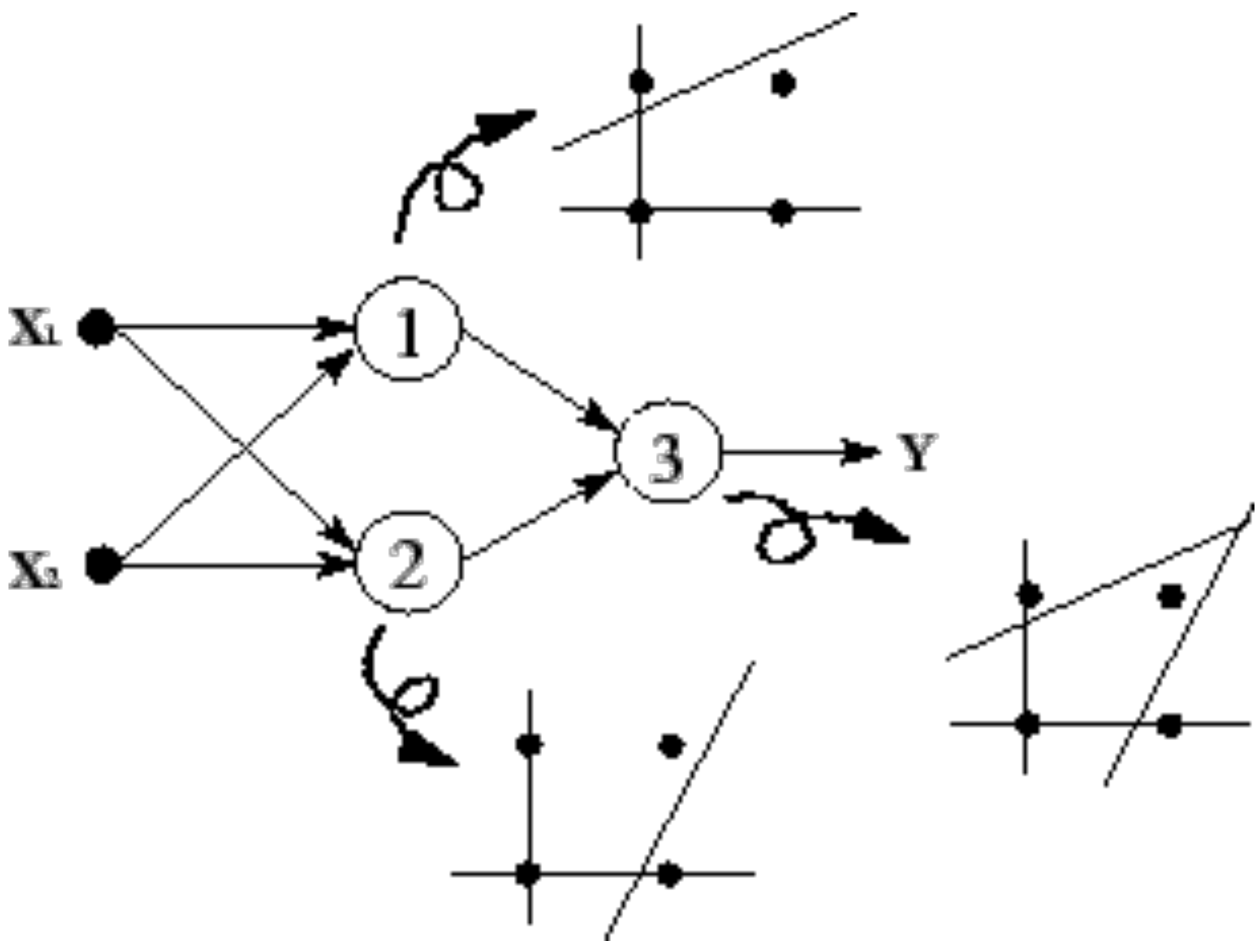

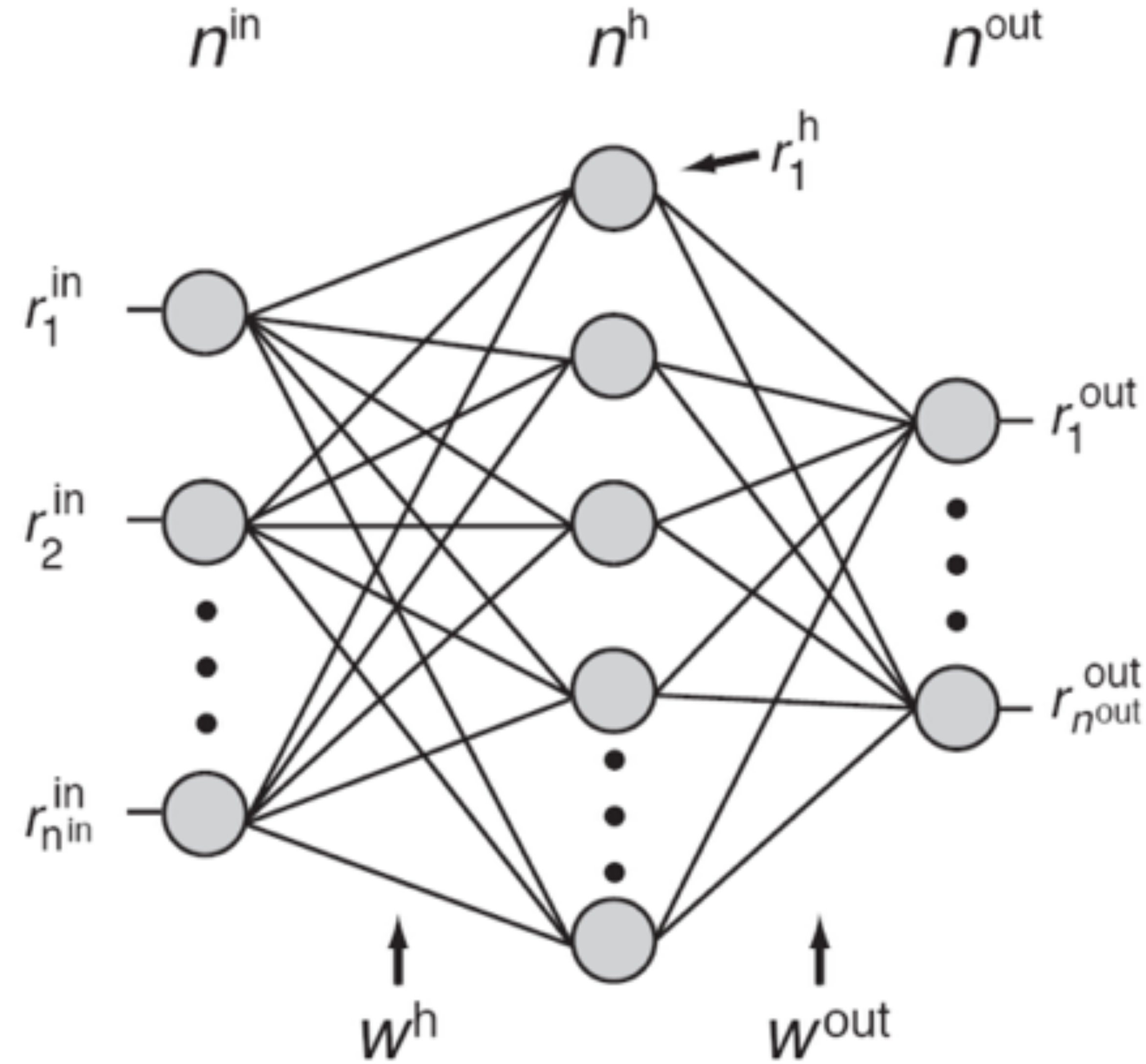

Trappenberg 2010

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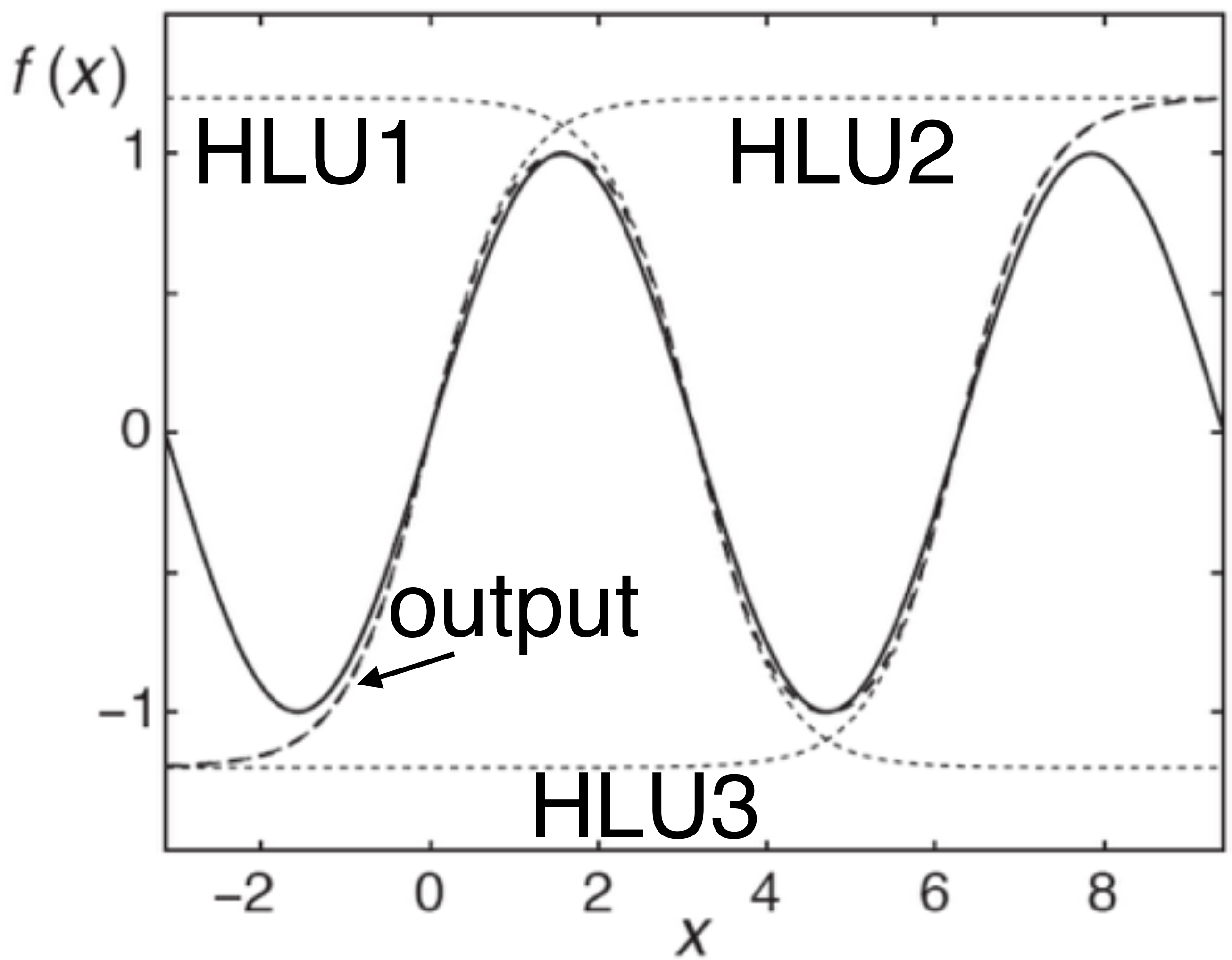
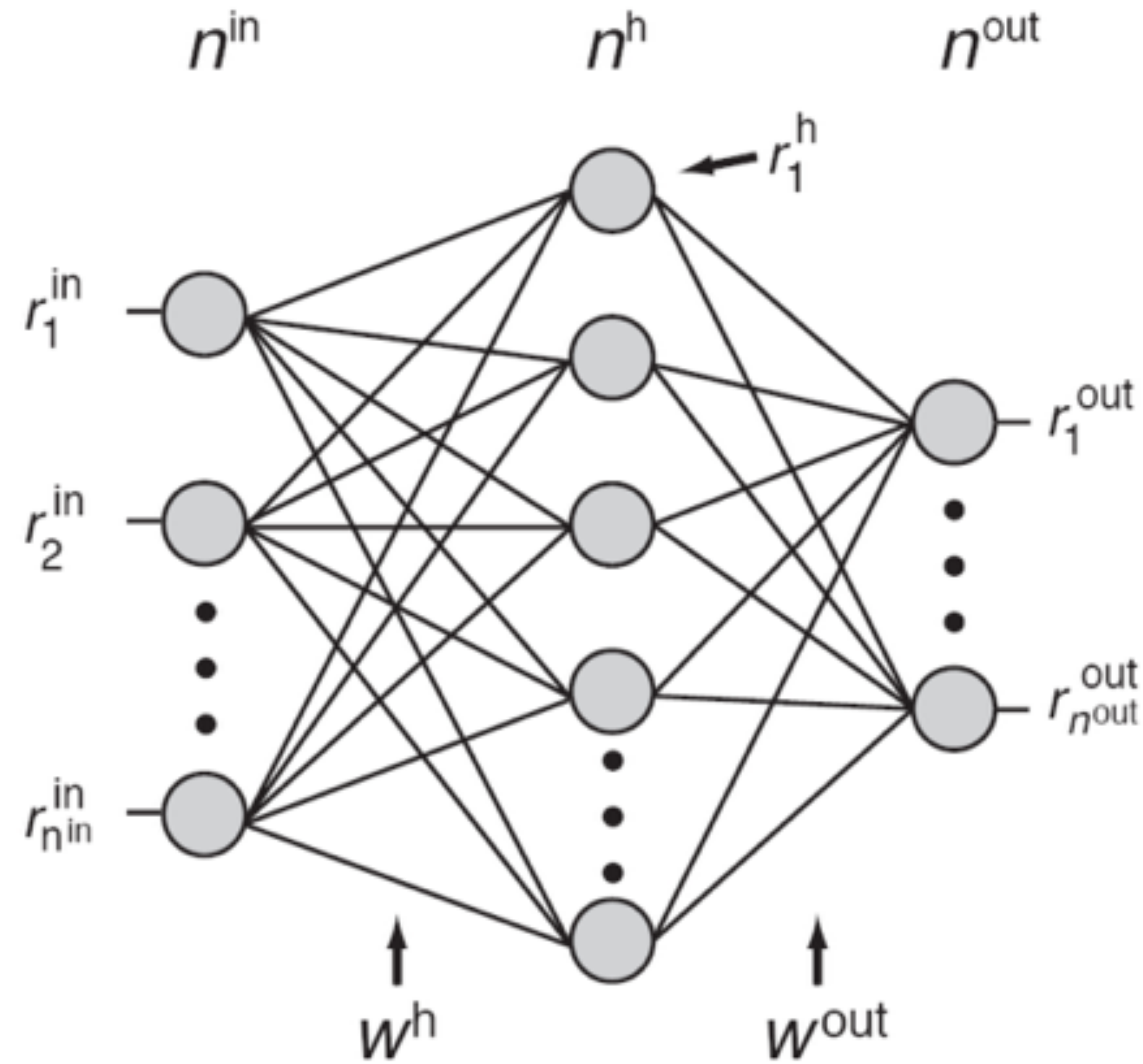
Multi-layer perceptron



E.g. sine wave approximation using logistic transfer function



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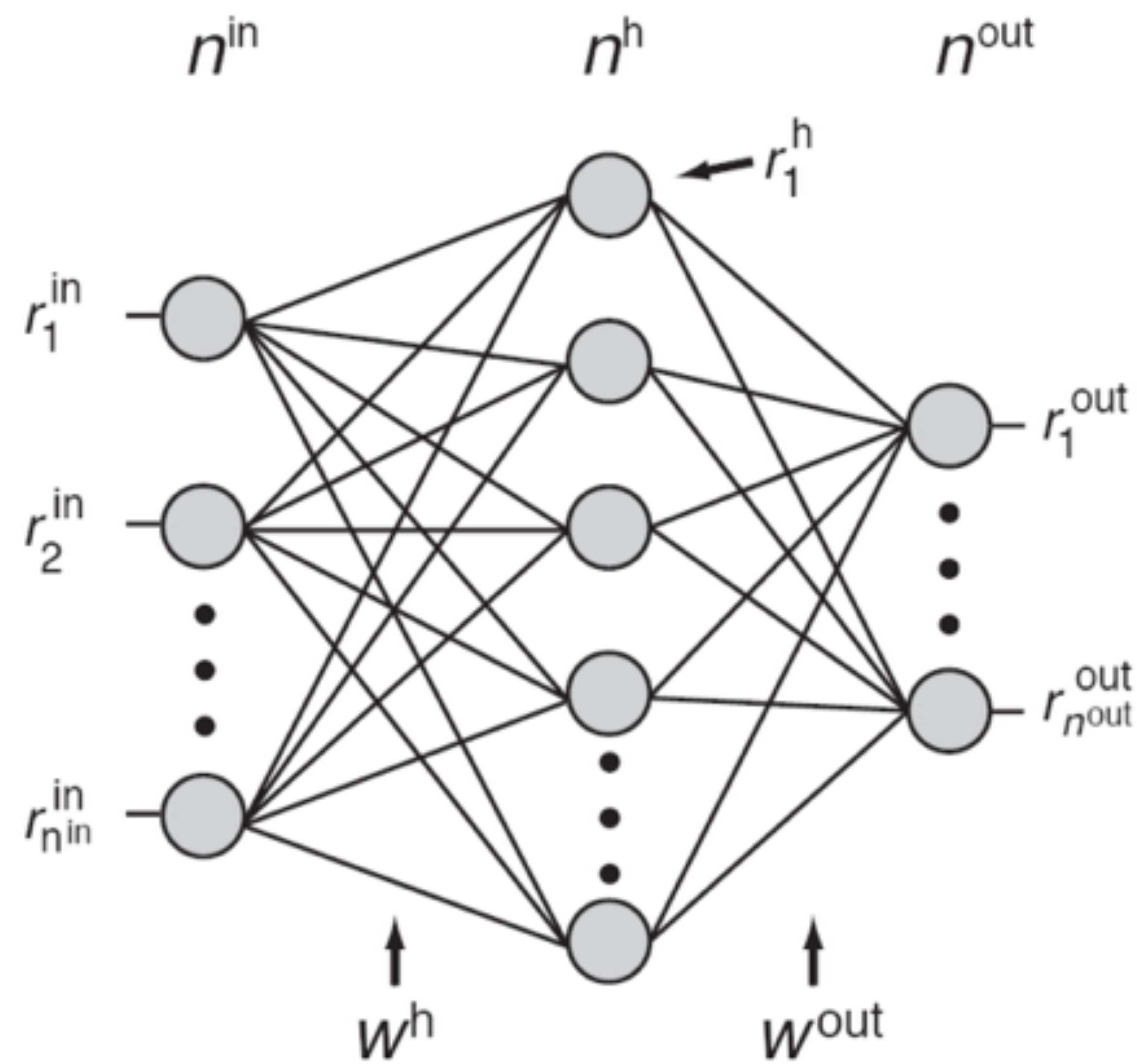
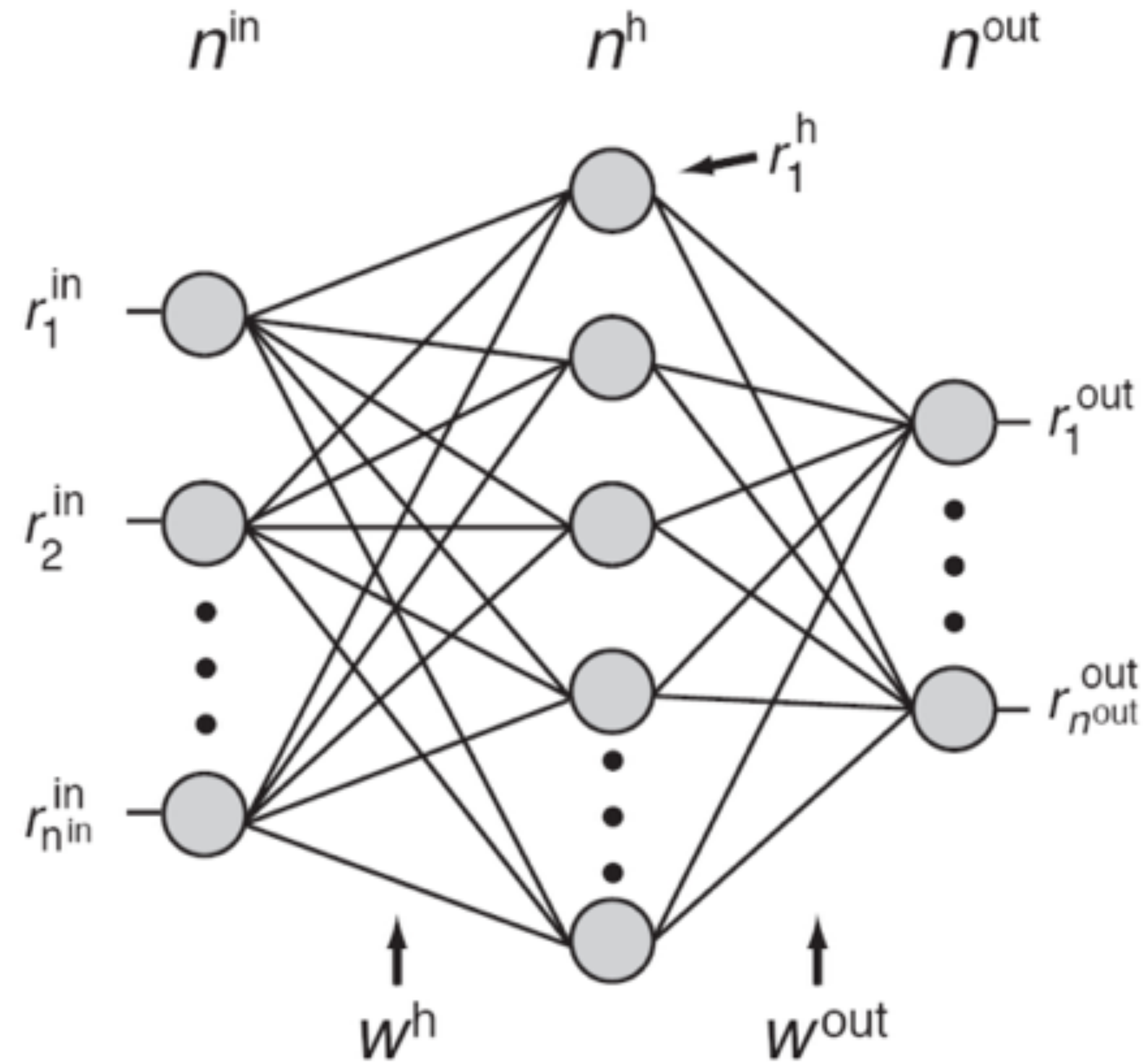


Trappenberg 2010

Multi-layer perceptron

Limitations



Trappenberg 2010

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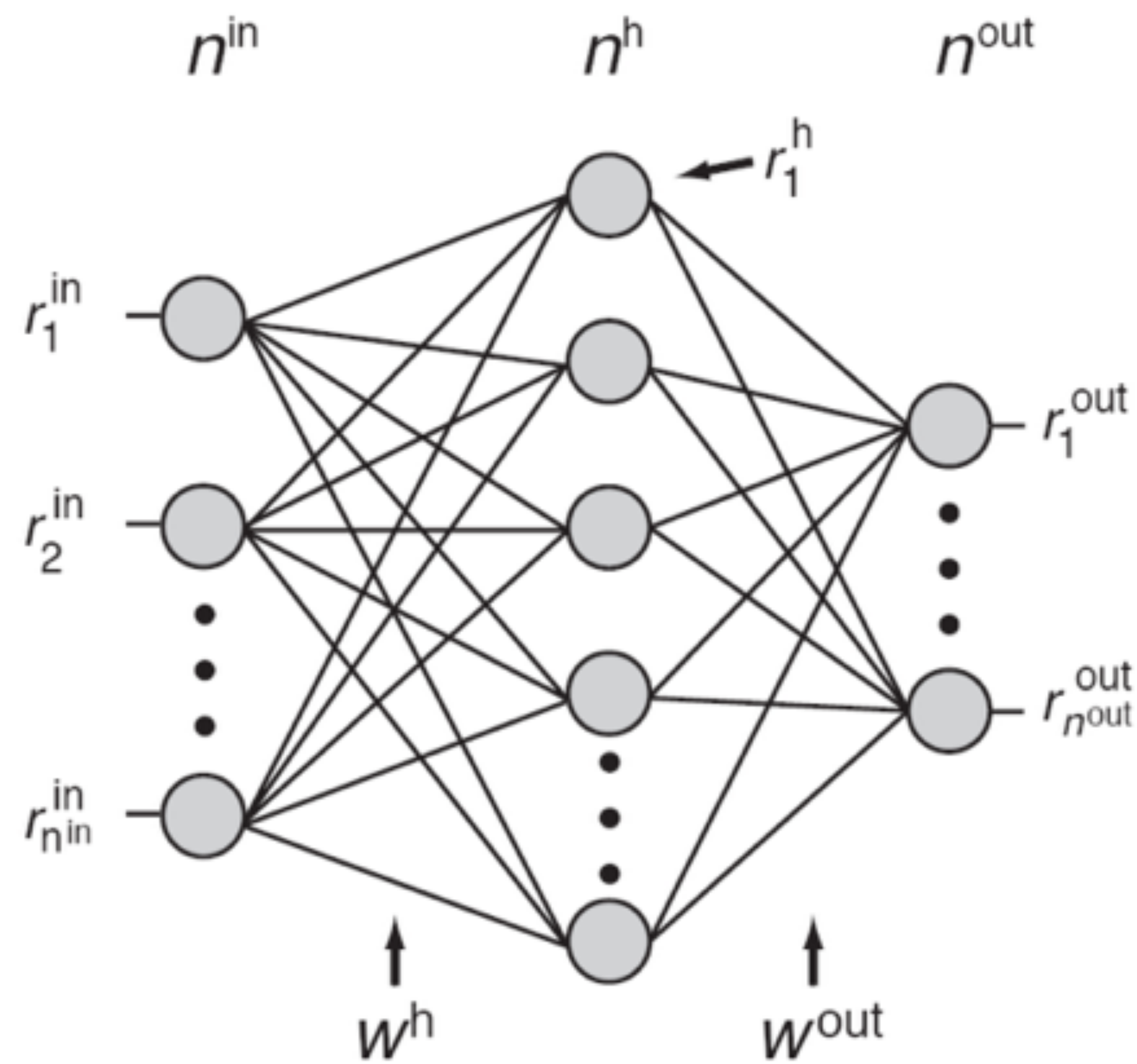

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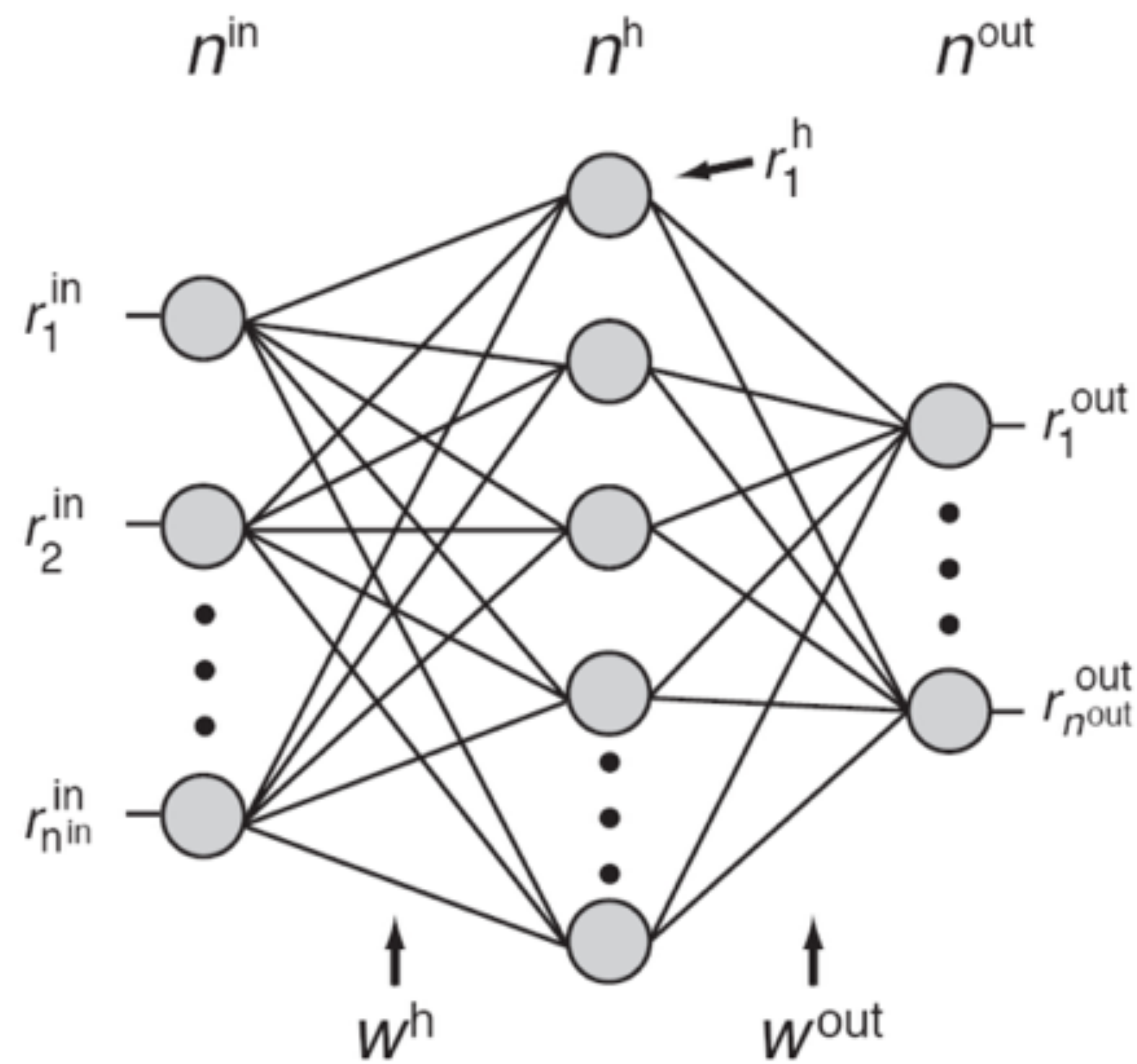

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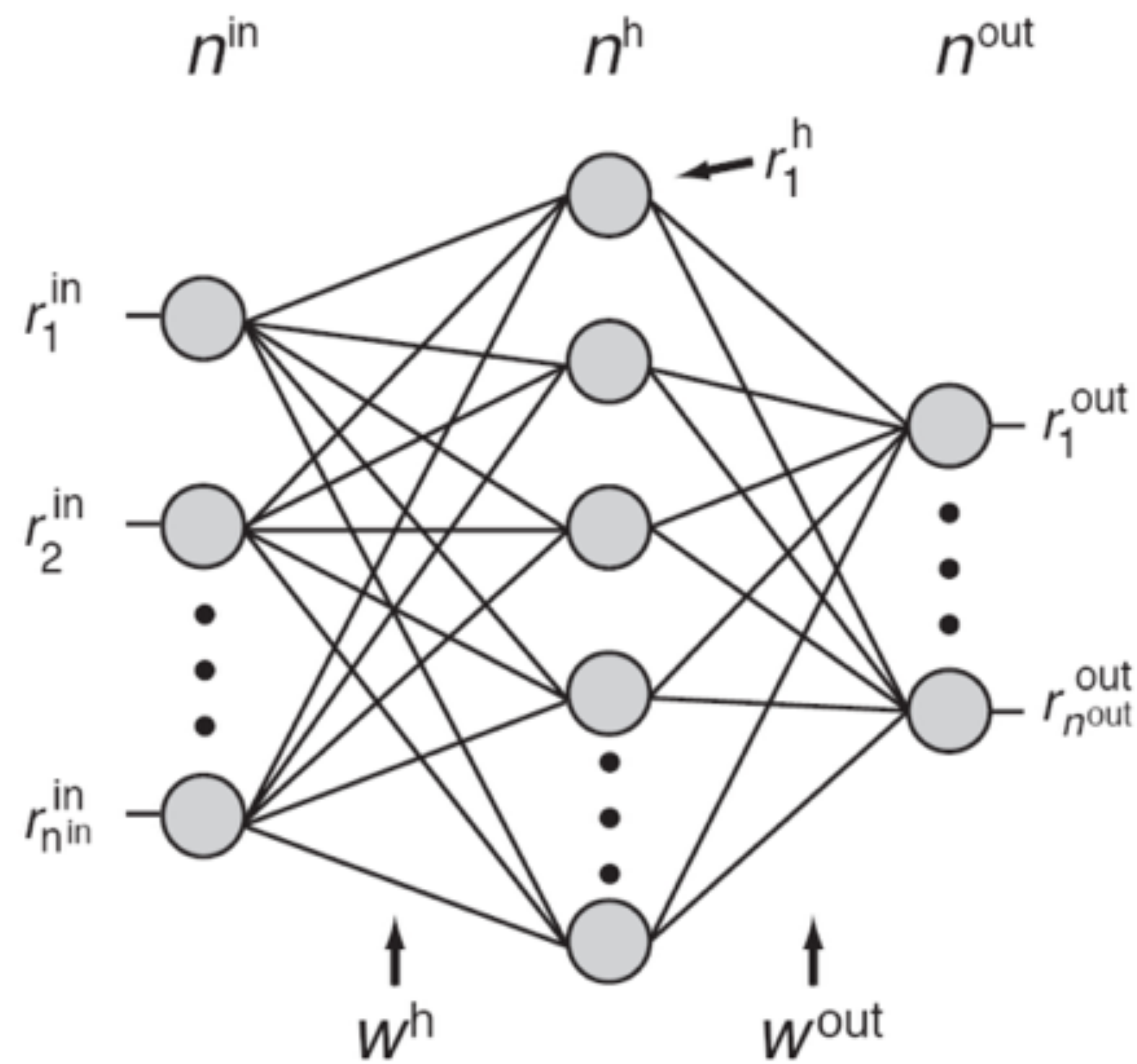

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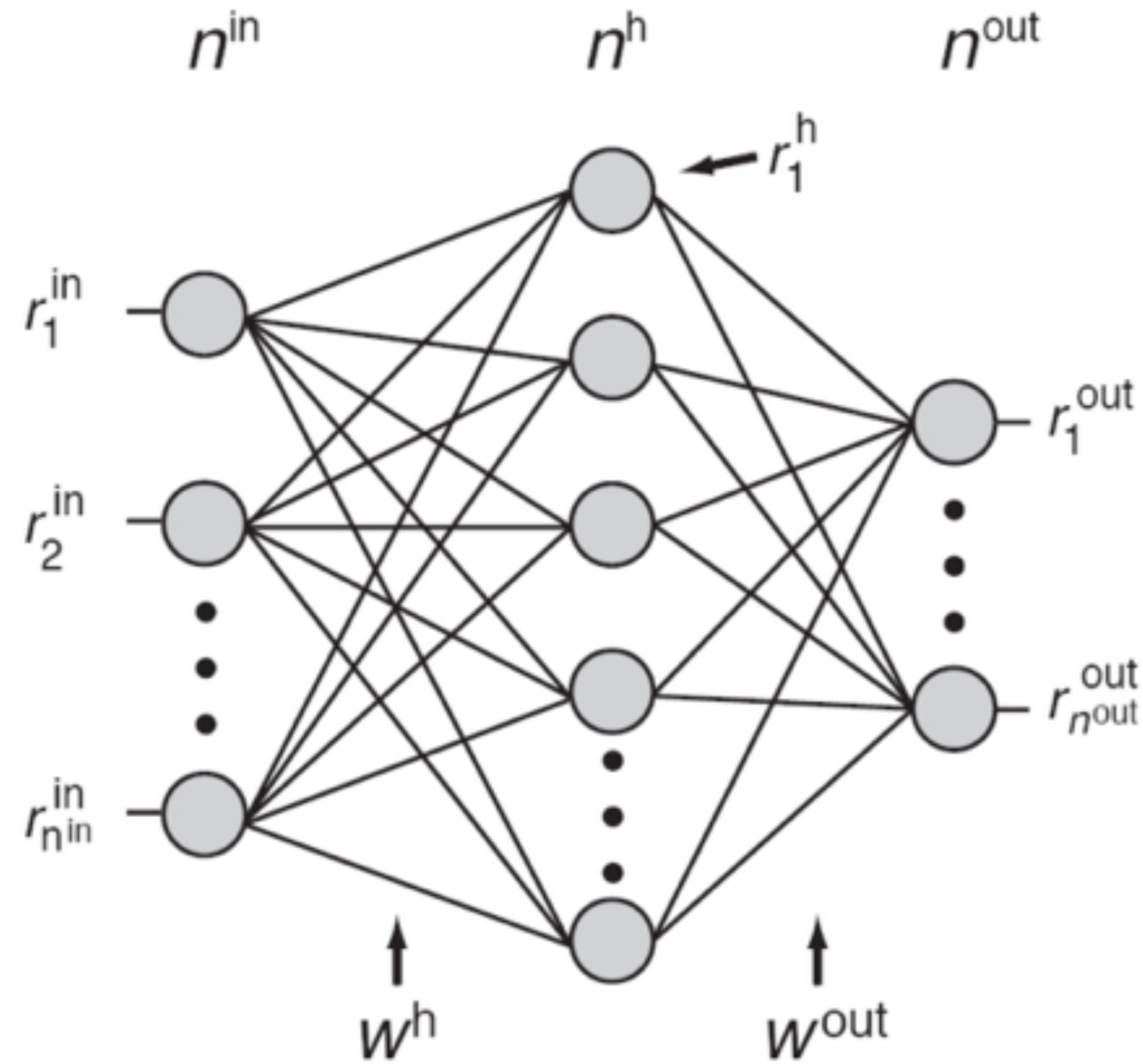

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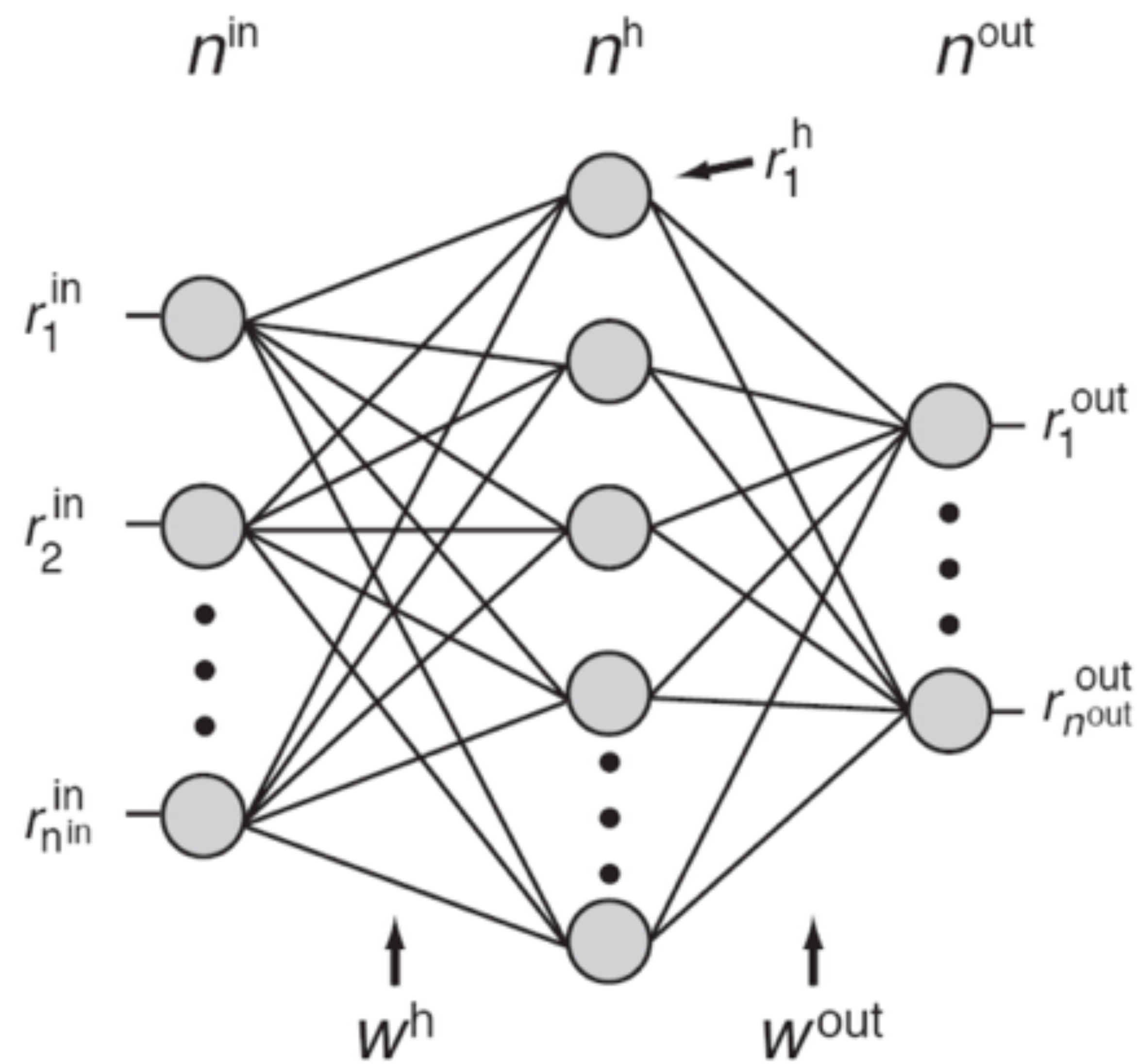

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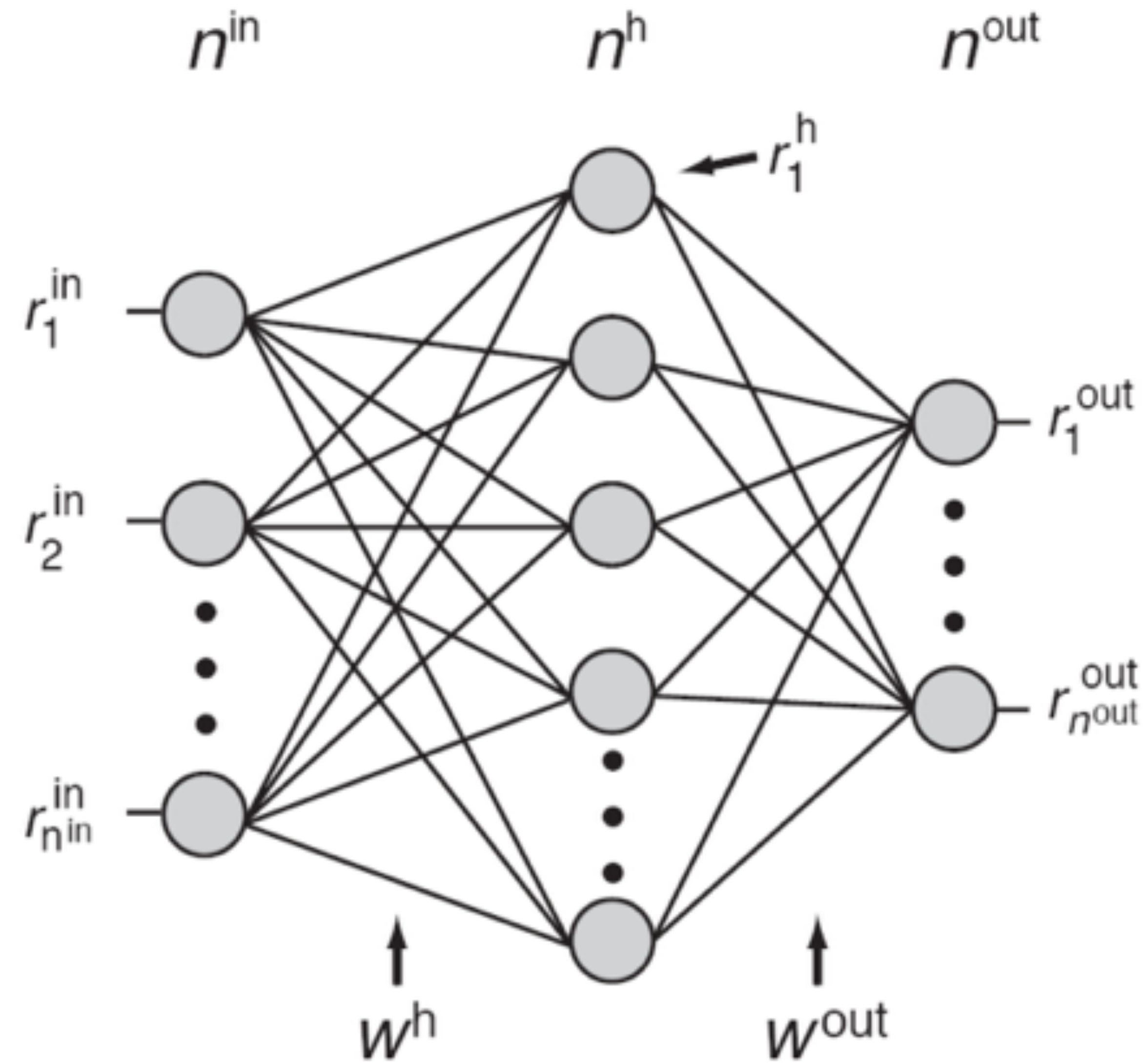

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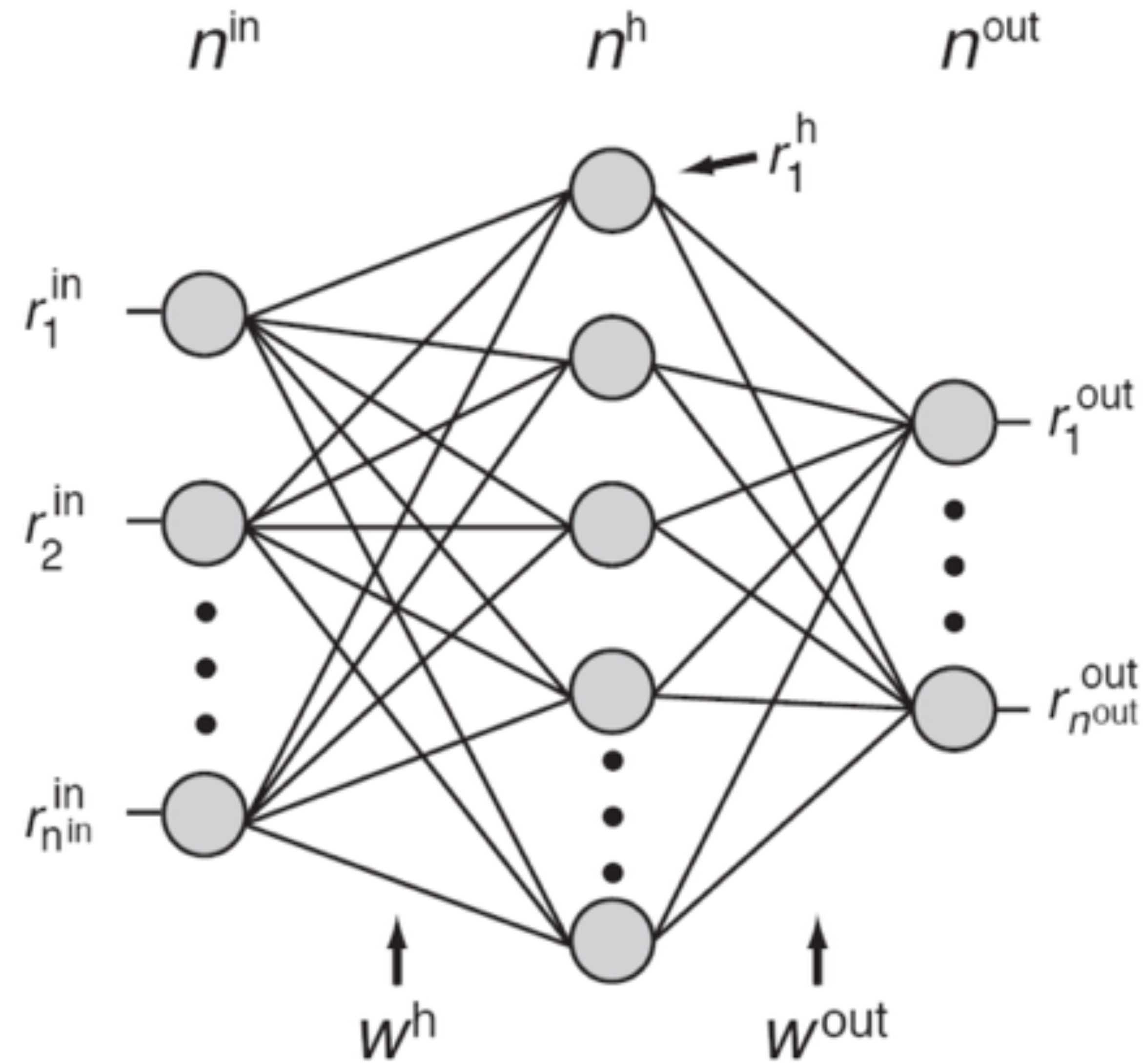

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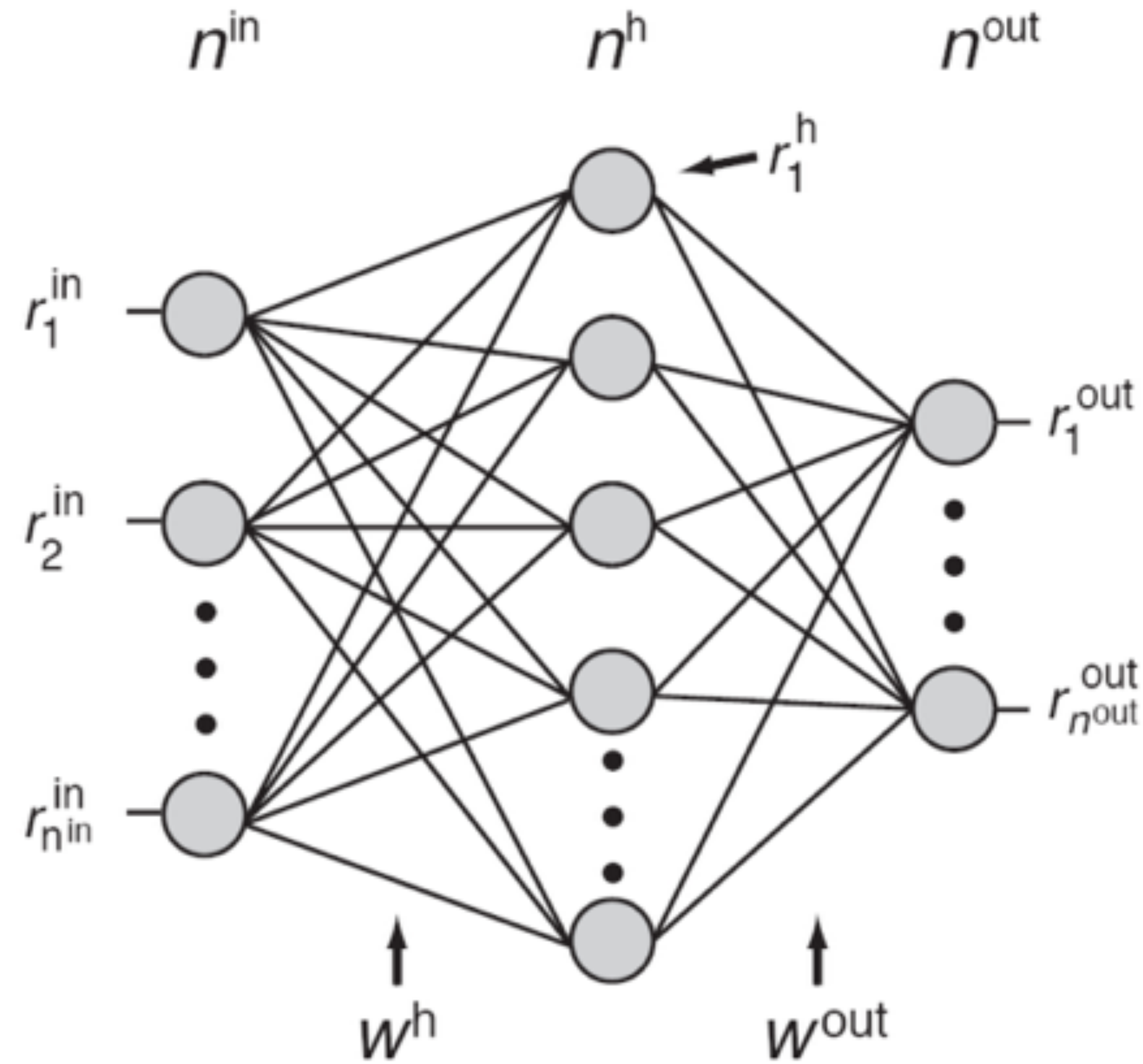

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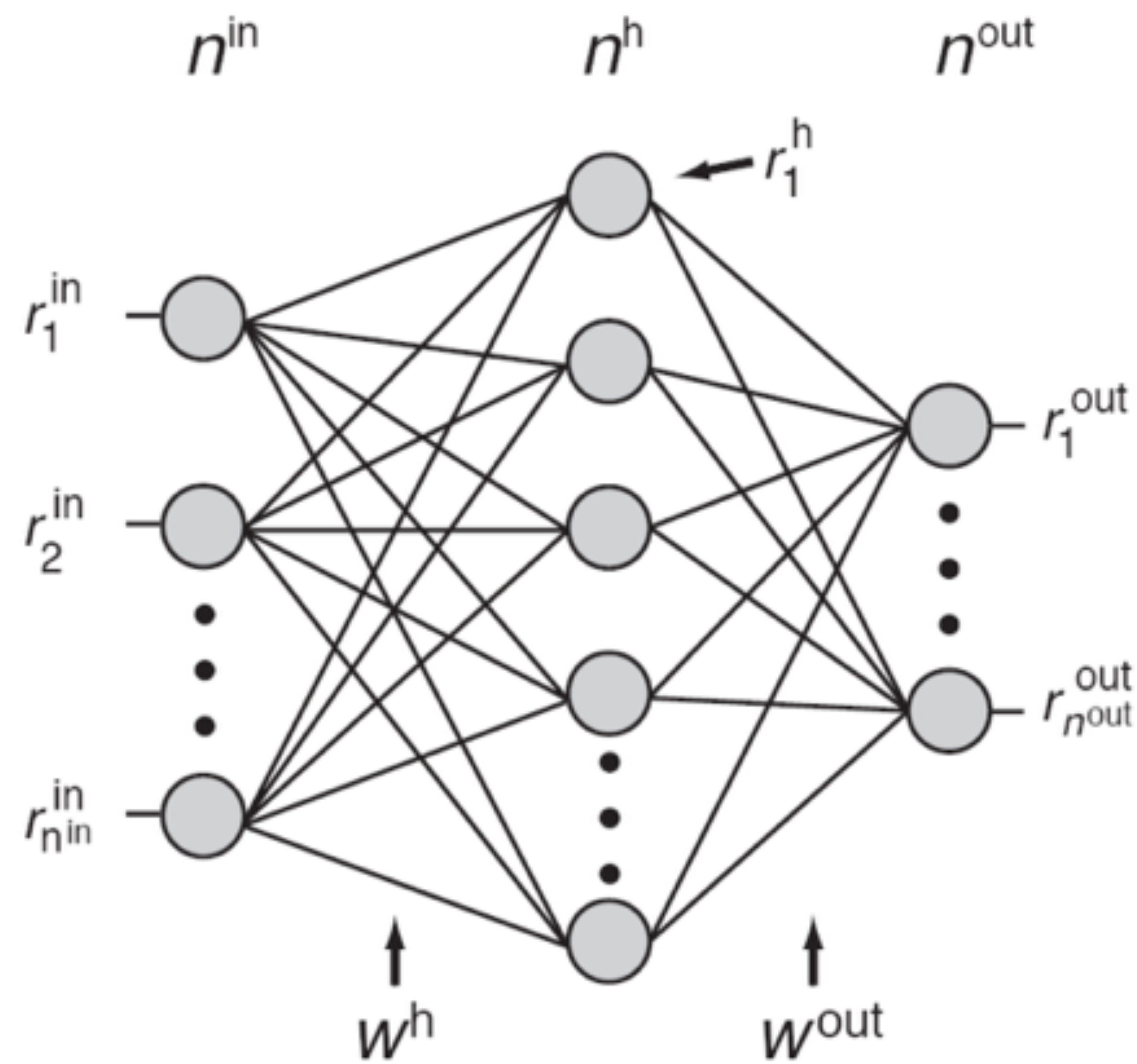


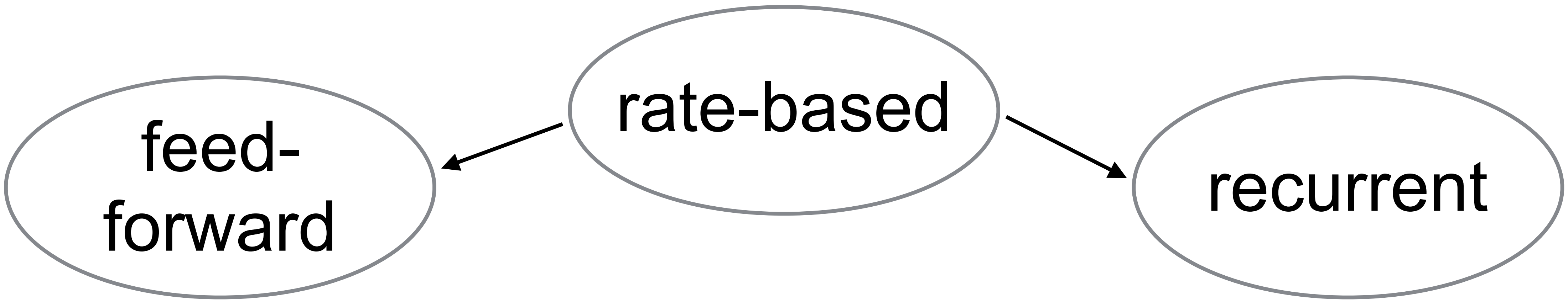
Trappenberg 2010

Point: MLP is usually good for machine learning purposes, but is not necessarily good for neuroscience theory all the time!

Two types of NNets

rate-based

Two types of NNets



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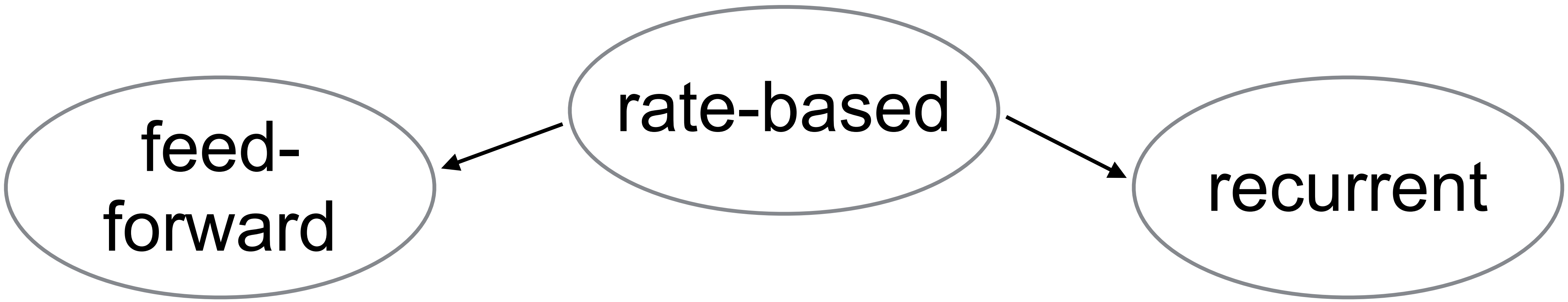
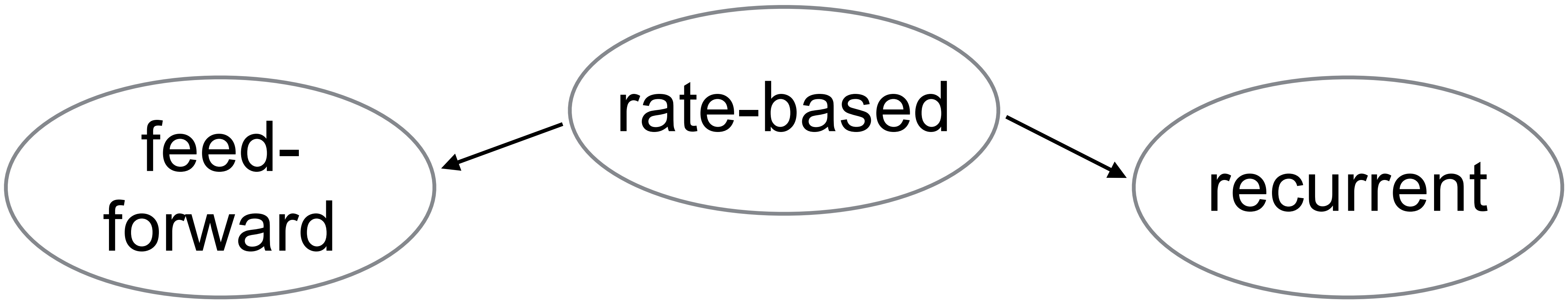


Feed-forward: information *only* flows forward, simplest connectivity

Two types of NNets



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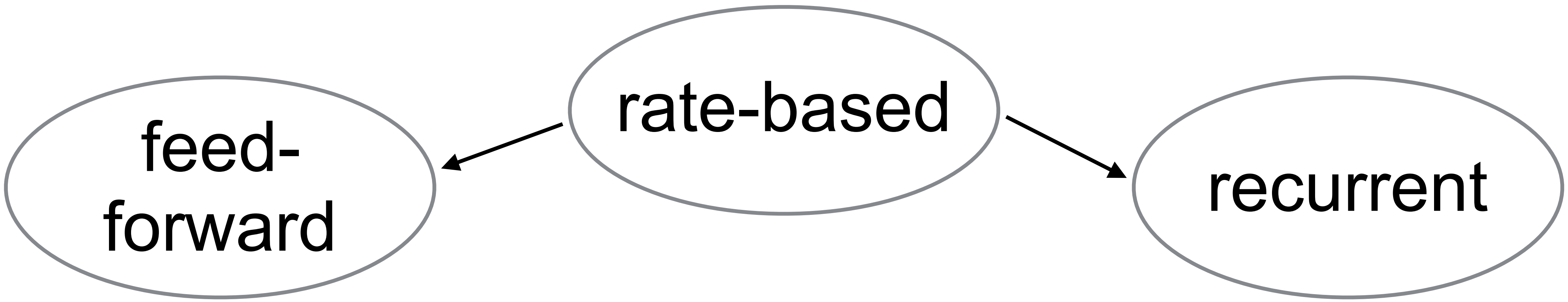
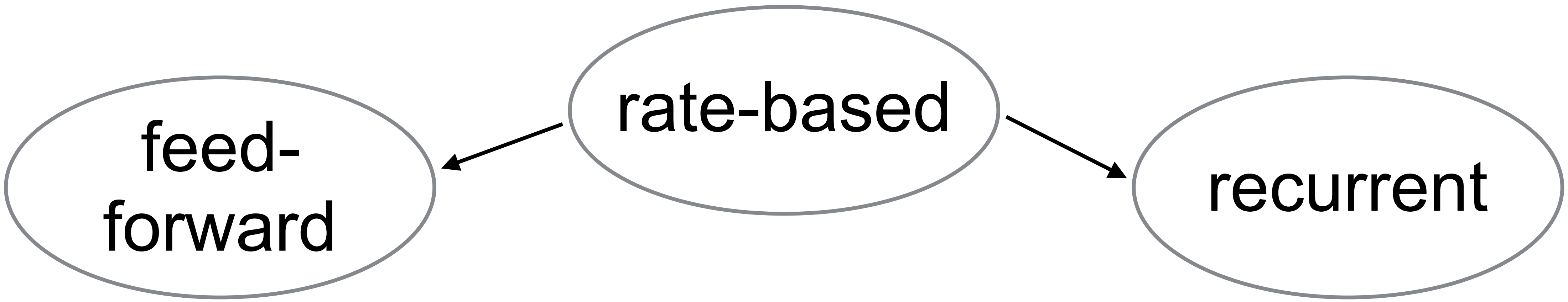


Feed-forward: information *only* flows forward, simplest connectivity

- Kernel machines

Two types of NNets



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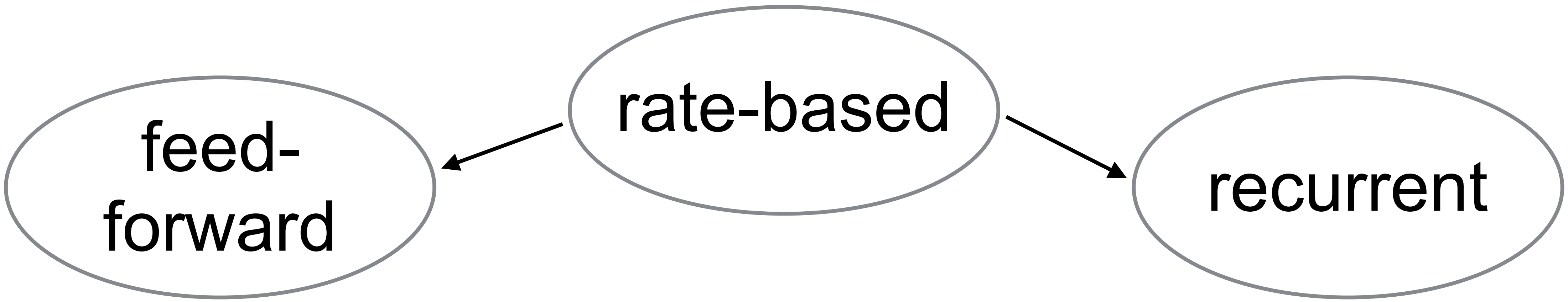
- Kernel machines
- Radial basis function networks

Two types of NNets



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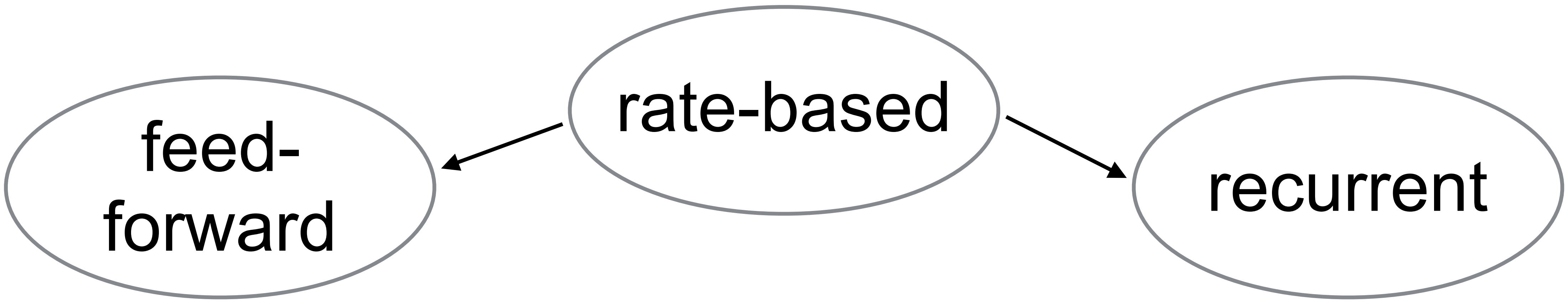

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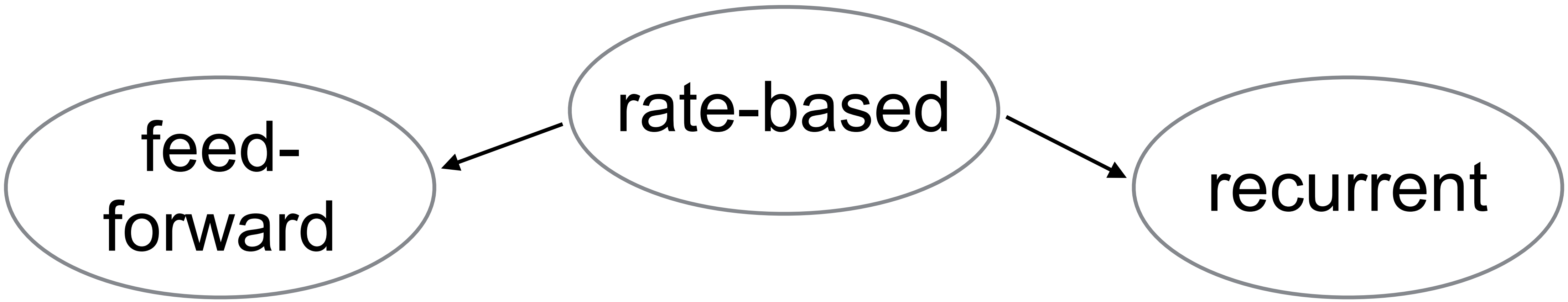
- Bayesian nets

Two types of NNets



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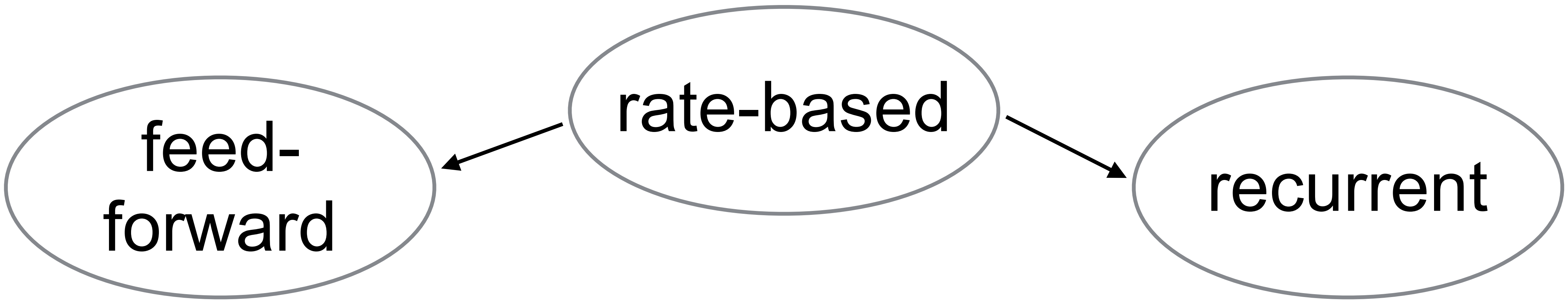

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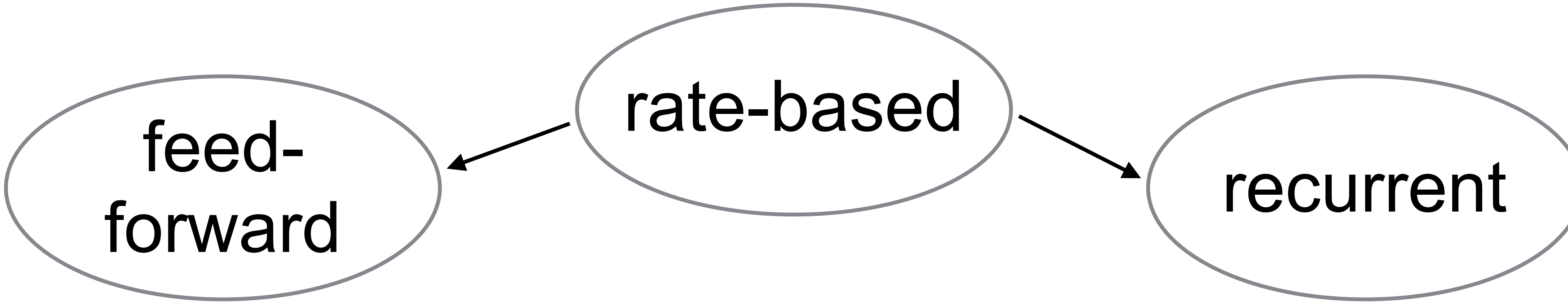

Feed-forward: information *only* flows forward, simplest connectivity

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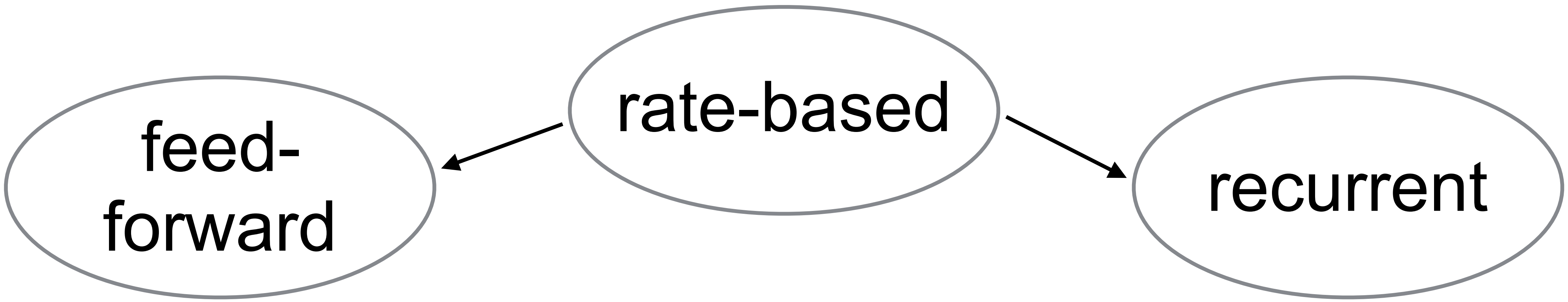
- Bayesian nets
- Stochastic nets
- Modular nets
- Committee machines

Two types of NNets



Feed-forward: information *only* flows forward, simplest connectivity

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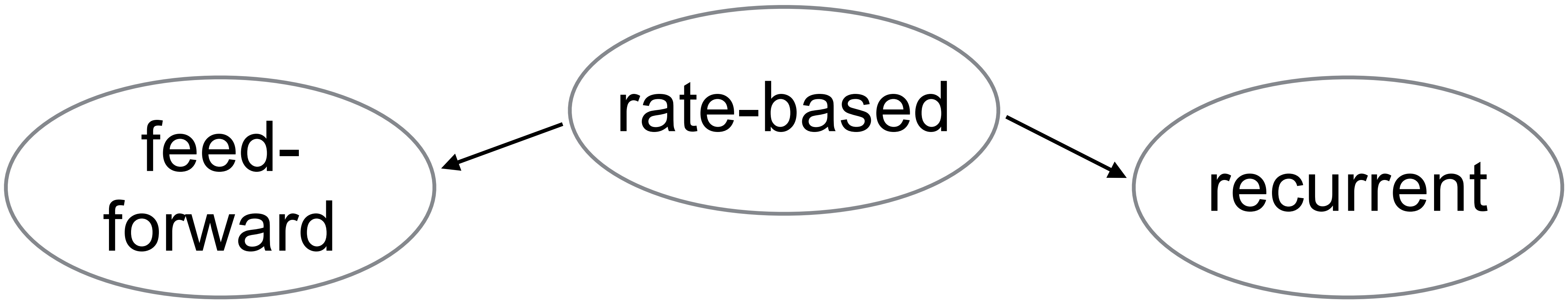
- Stochastic nets
- Modular nets
- Committee machines
- Associative neural nets

Two types of NNets



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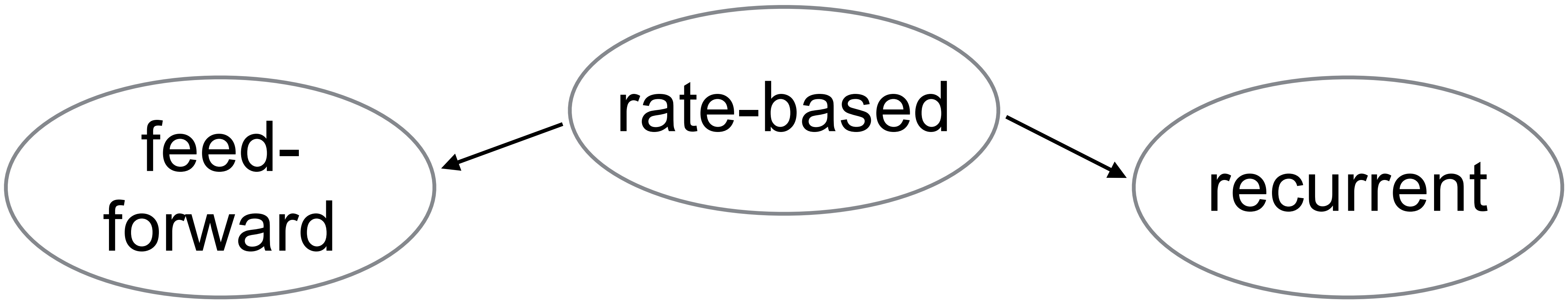
- Modular nets
- Committee machines
- Associative neural nets
- Holographic associative nets (???)

Two types of NNets



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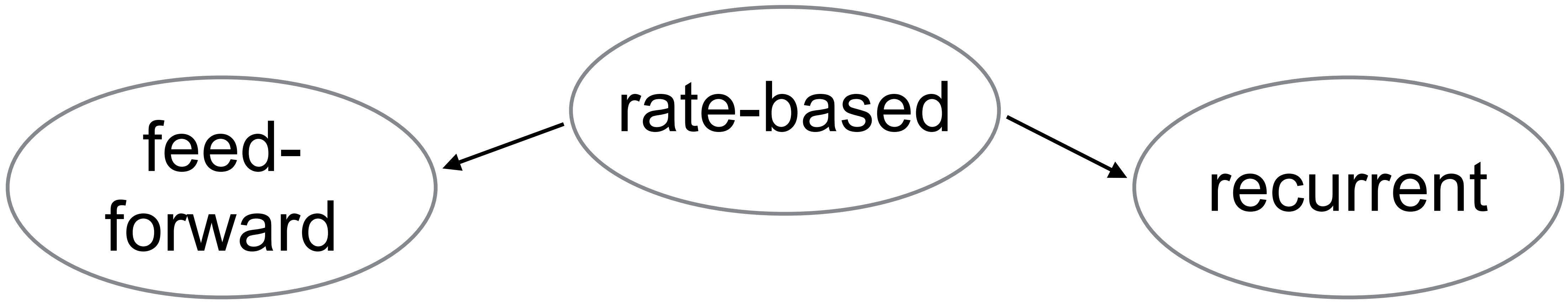
- Fuzzy nets

Two types of NNets



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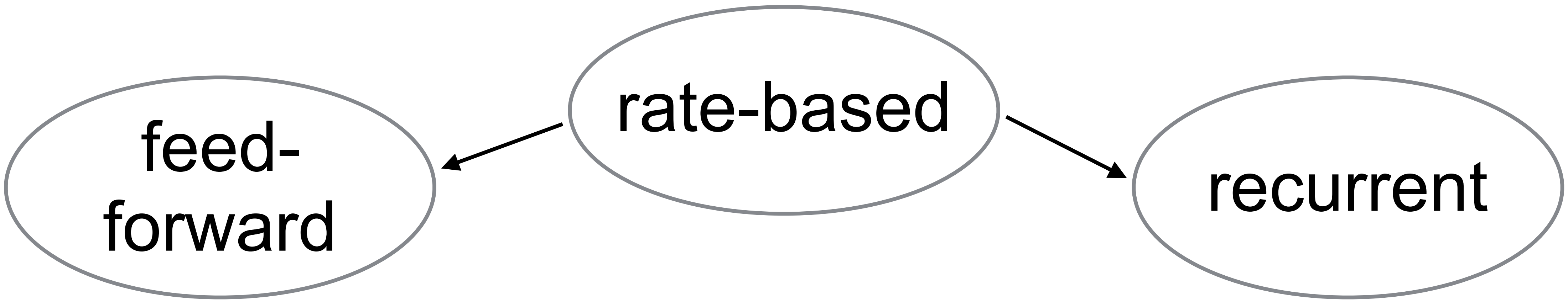
- Stochastic nets
- Modular nets
- Committee machines
- Associative neural nets
- Holographic associative nets (???)
- Fuzzy nets
- Compositional pattern-producing nets

Two types of NNets



Feed-forward: information *only* flows forward, simplest connectivity

- Kernel machines
- Radial basis function networks
- Probabilistic mapping networks
- Bayesian nets
- Stochastic nets
- Modular nets
- Committee machines
- Associative neural nets
- Holographic associative nets (???)
- Fuzzy nets
- Compositional pattern-producing nets
- etc.

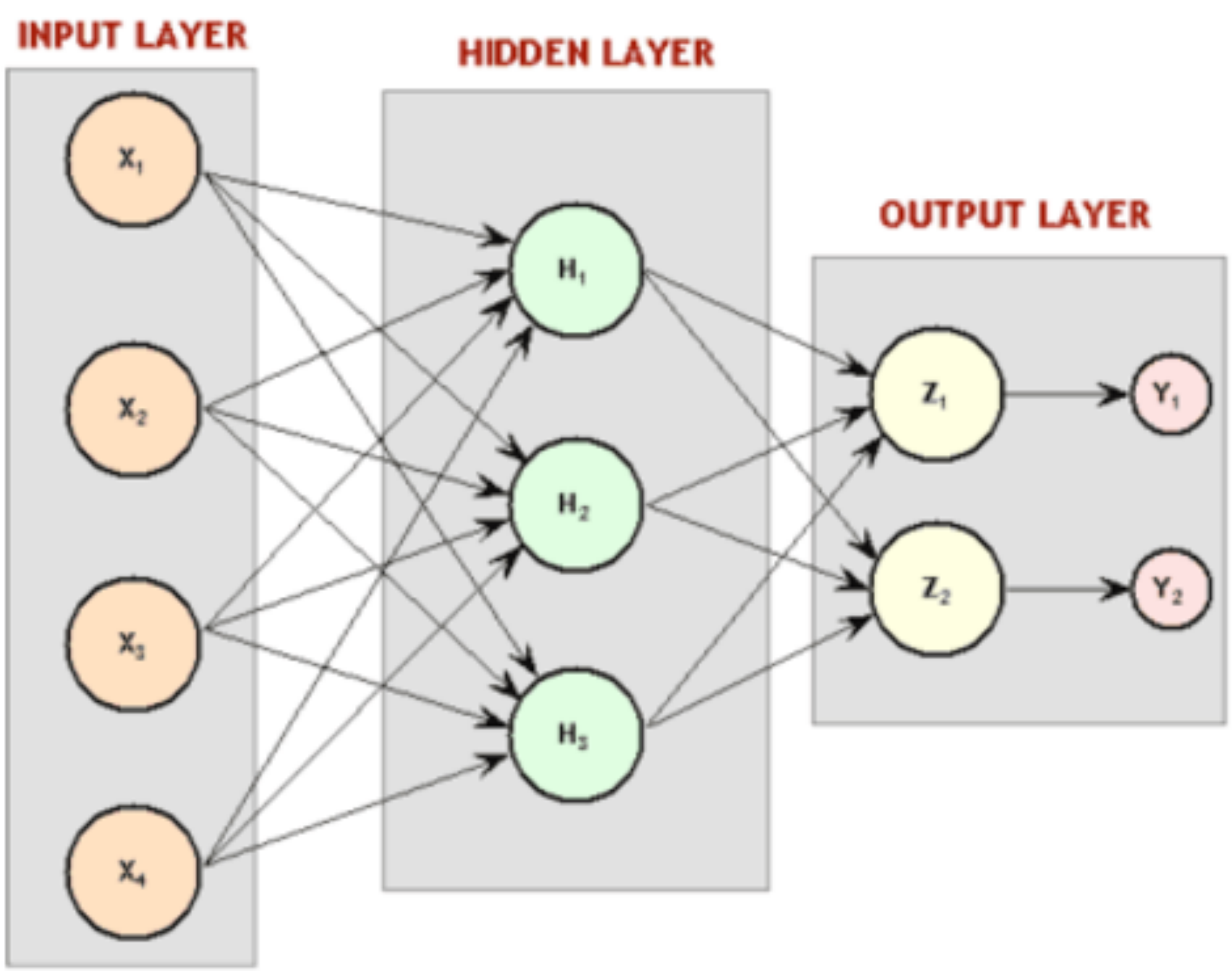
Two types of NNets

rate-based

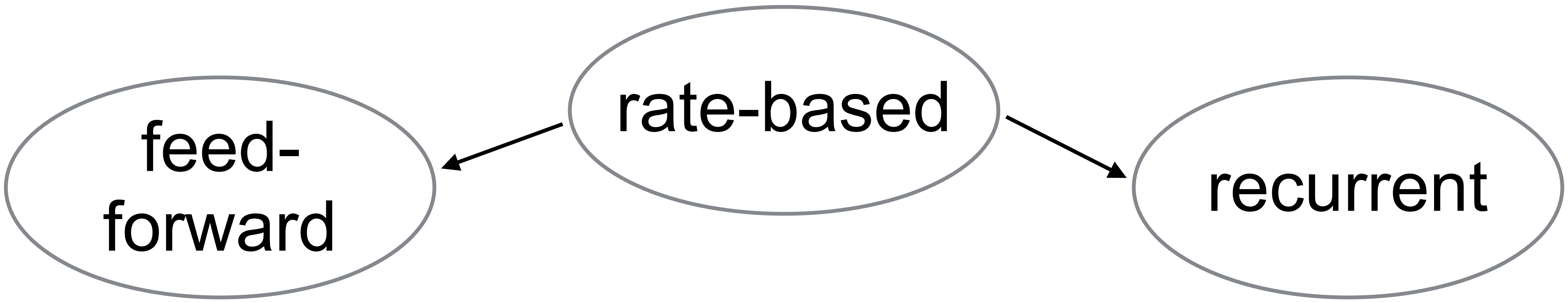
feed-forward

recurrent

Feed-forward: information *only* flows forward, simplest connectivity

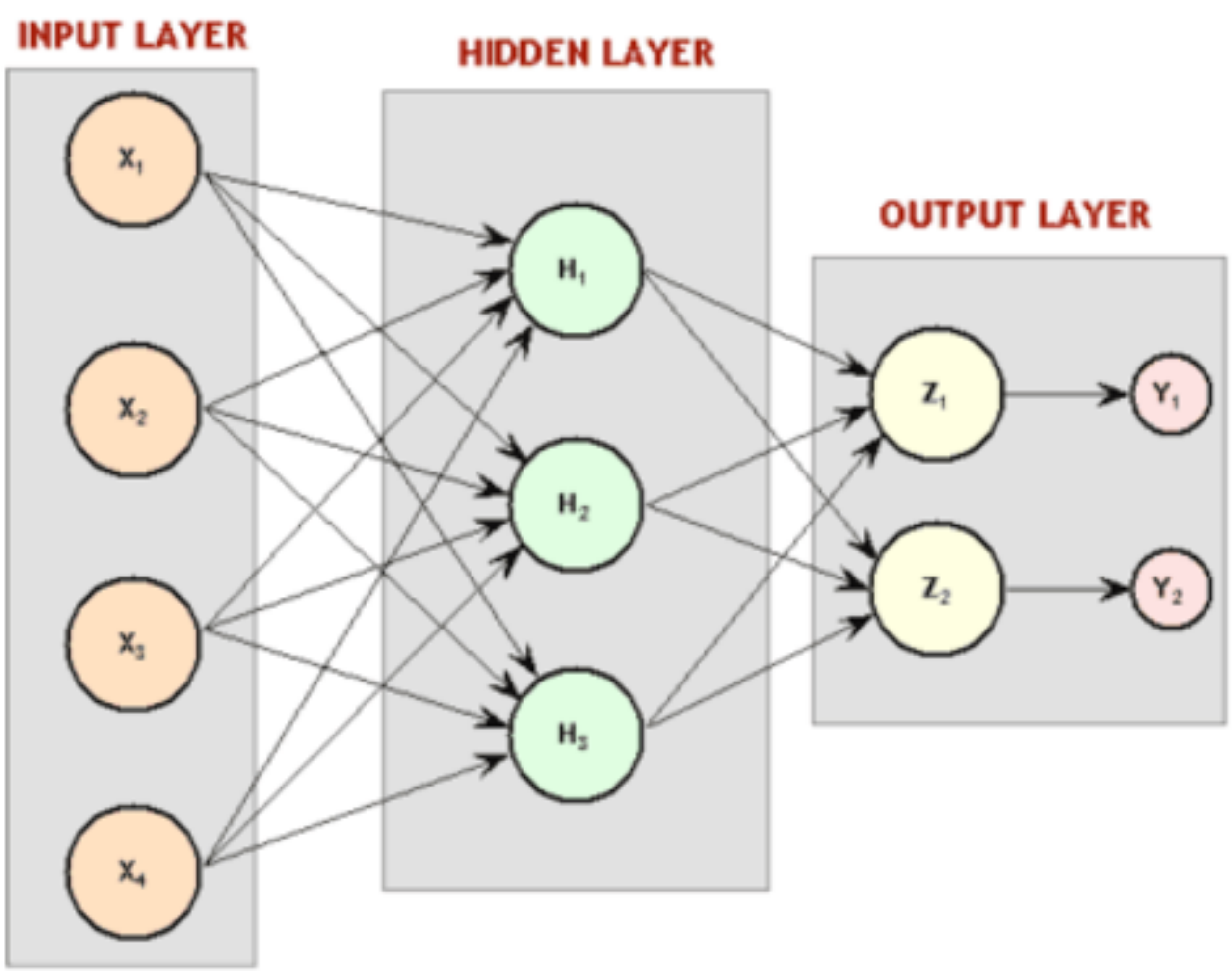


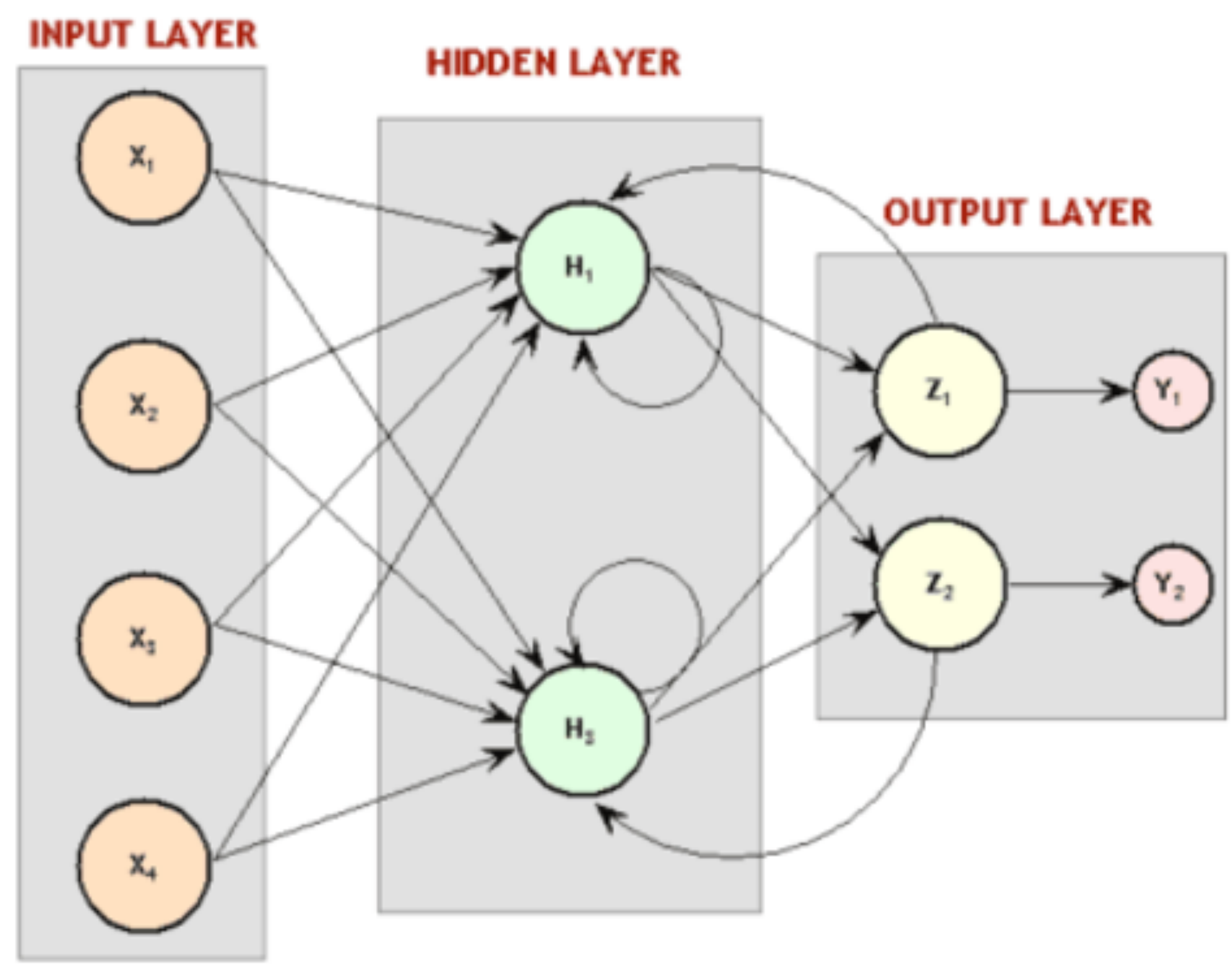
Two types of NNets



Feed-forward: information *only* flows forward, simplest connectivity

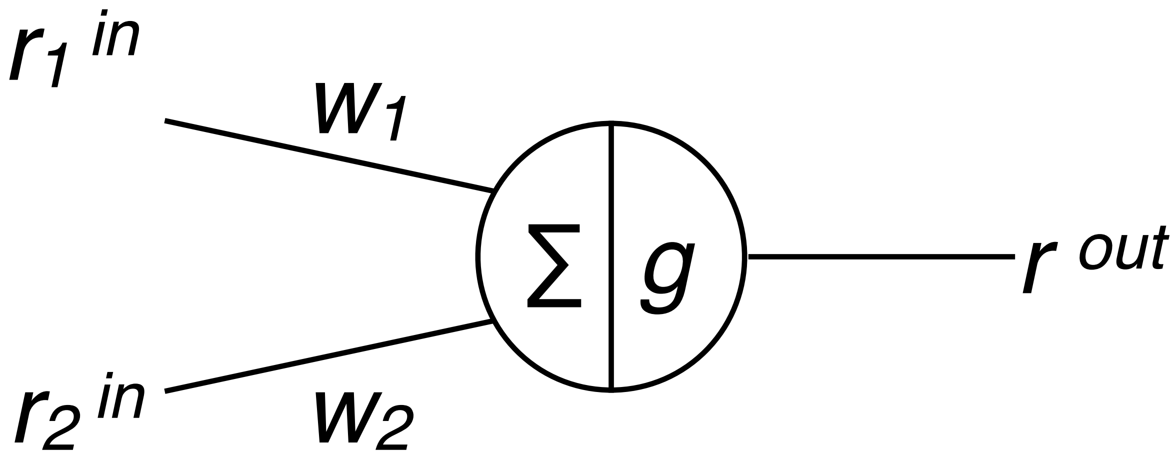
Recurrent: information can flow forward and backward, useful for time-resolved problems





How do we train a neural network?

Minimize some cost function… gradient descent!



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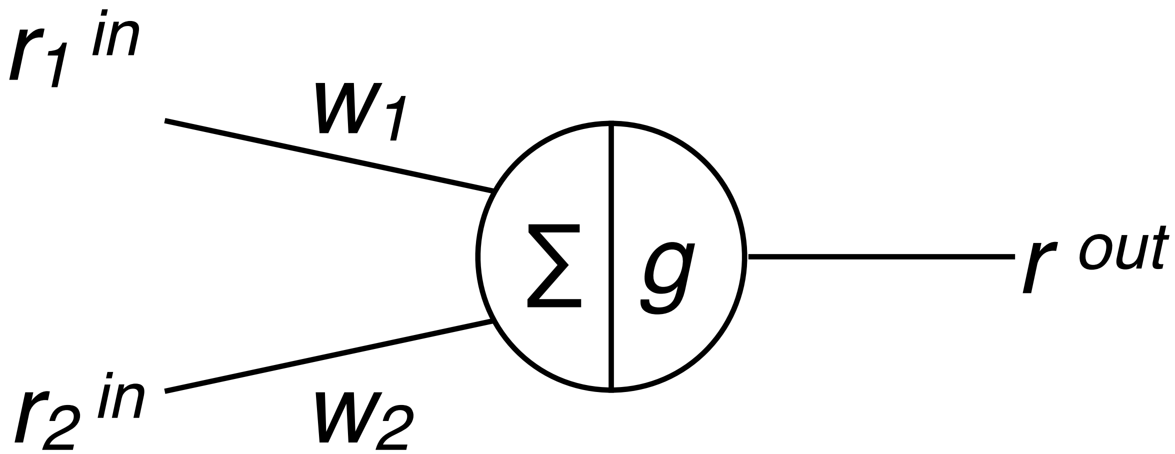

MSE

$$E = \frac{1}{2}\sum_i \left(r_i^{out} - y_i\right)^2$$

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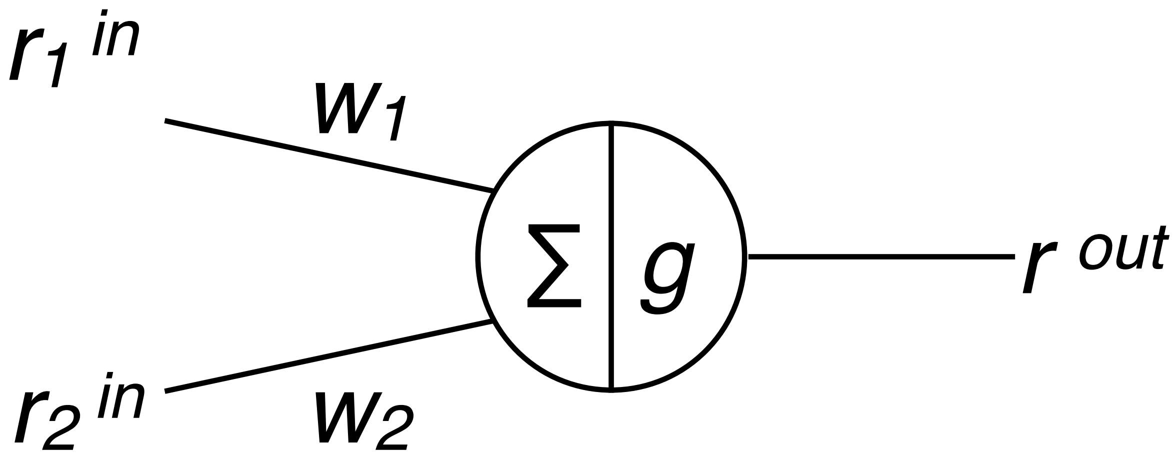

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desired output
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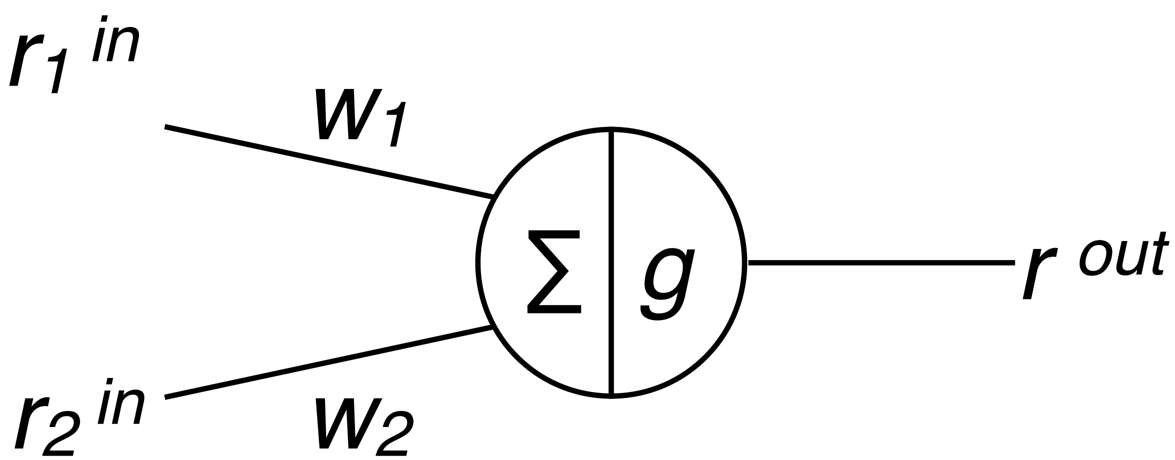

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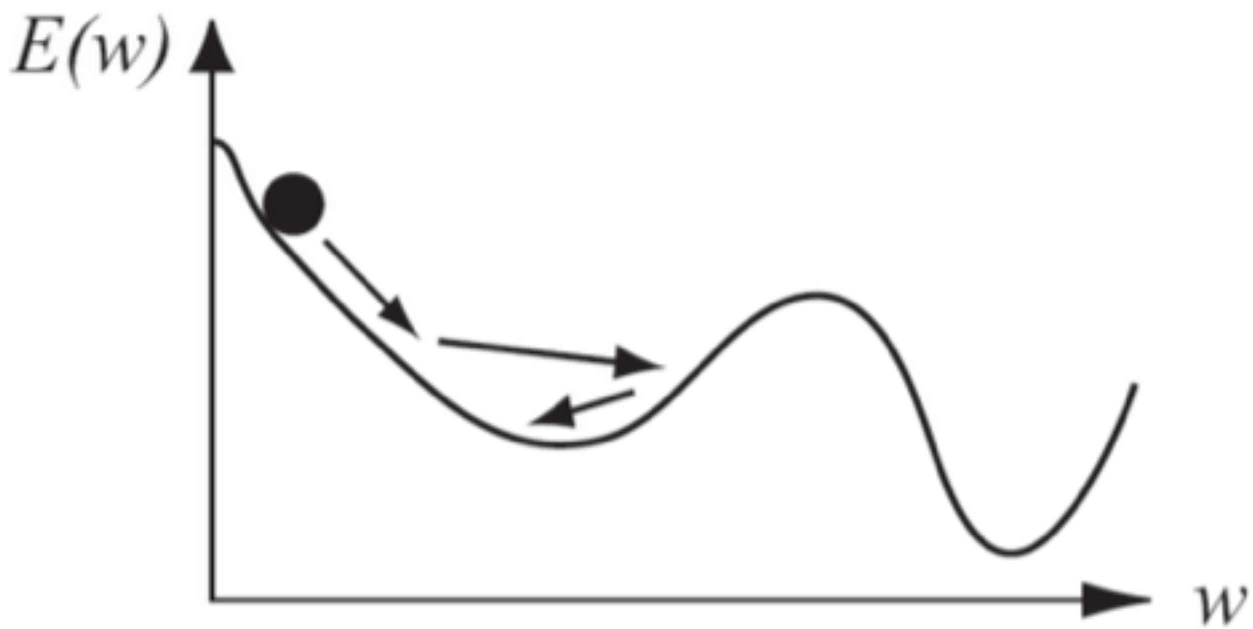

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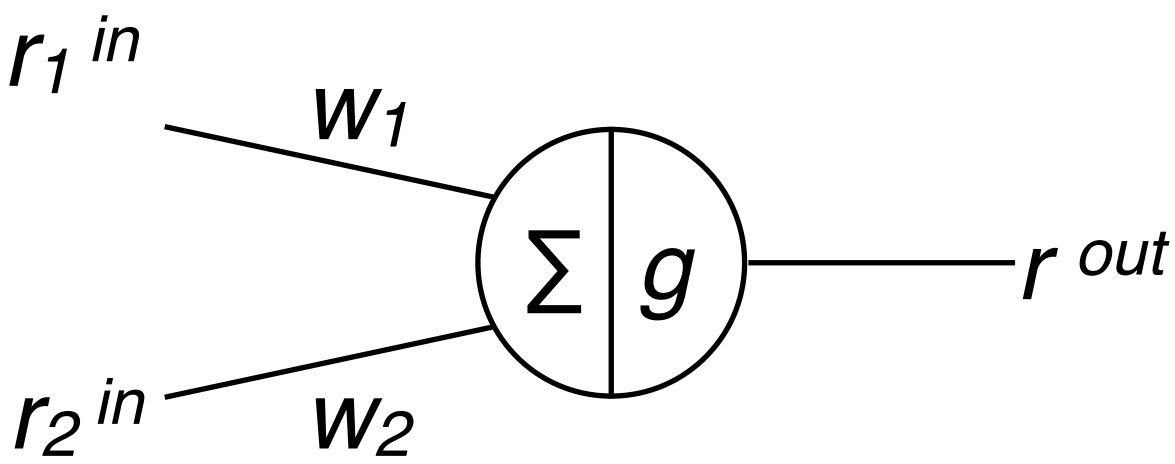

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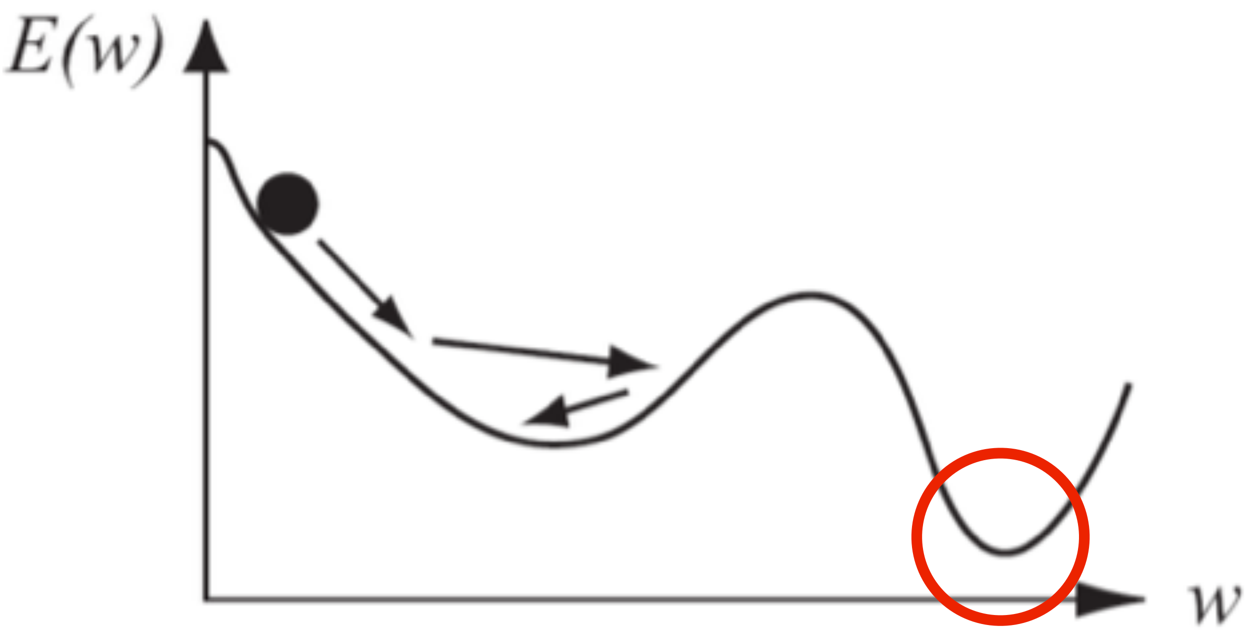

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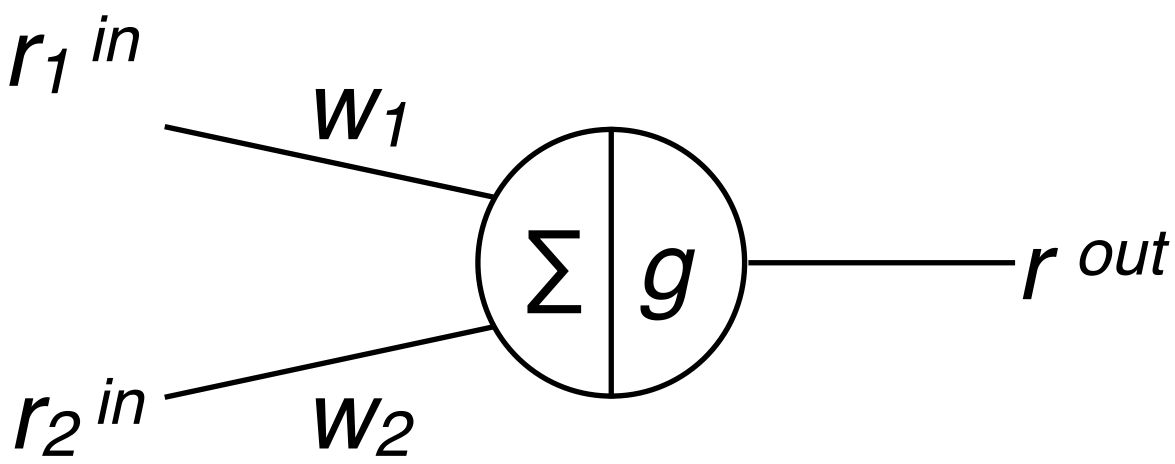

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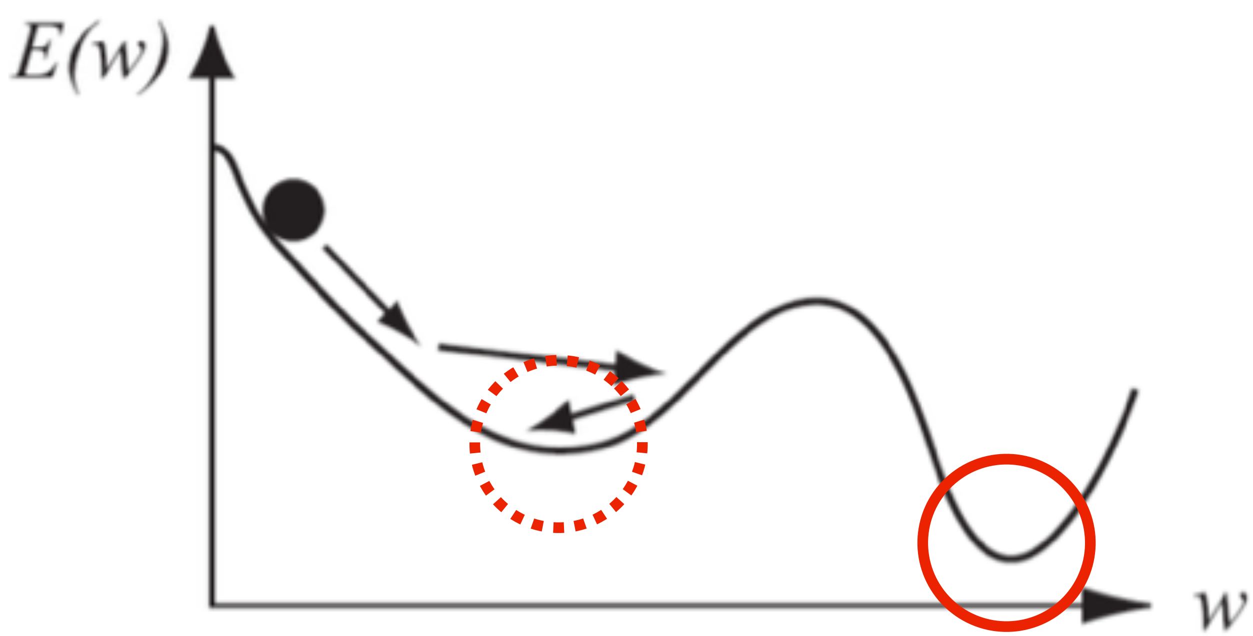

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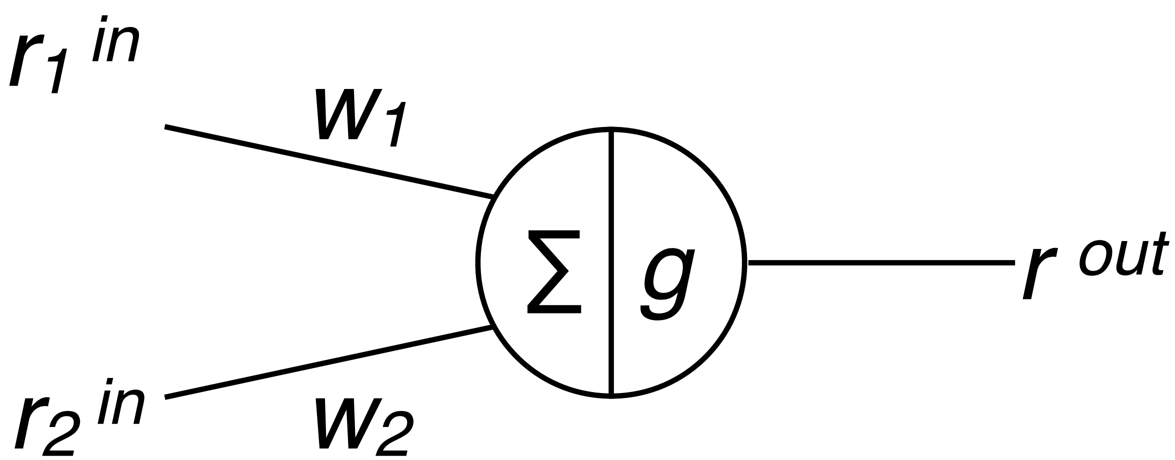

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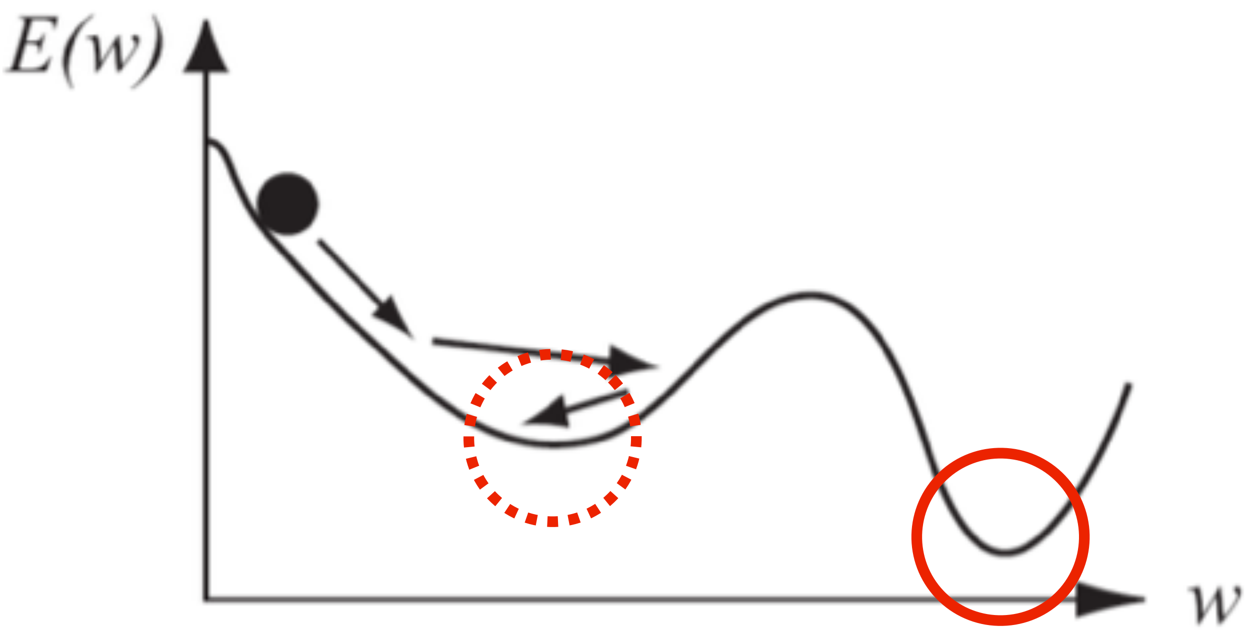

$$w_{ij} \leftarrow w_{ij} + \Delta w_{ij}$$

$$\Delta w_{ij} = -\varepsilon \left(\frac{\partial E}{\partial w_{ij}}\right)$$

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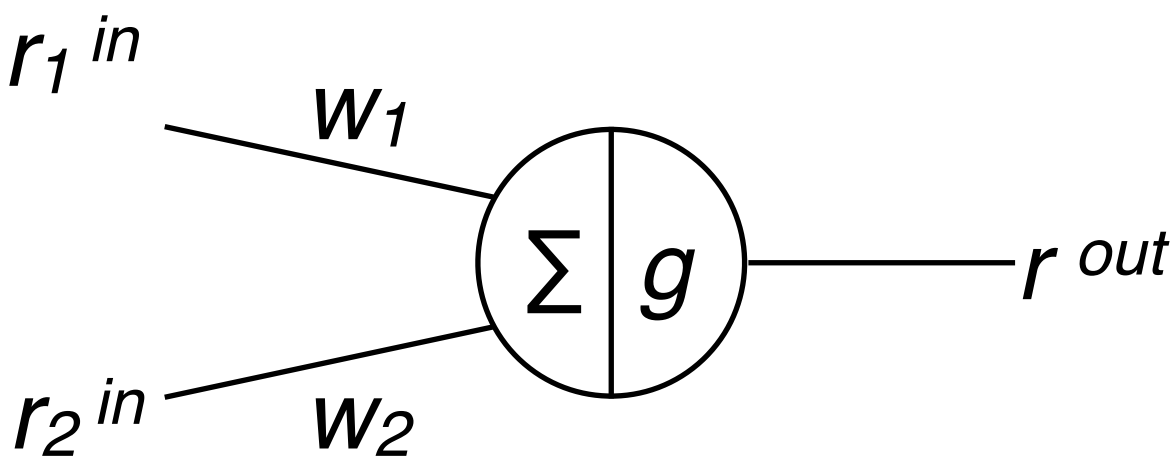

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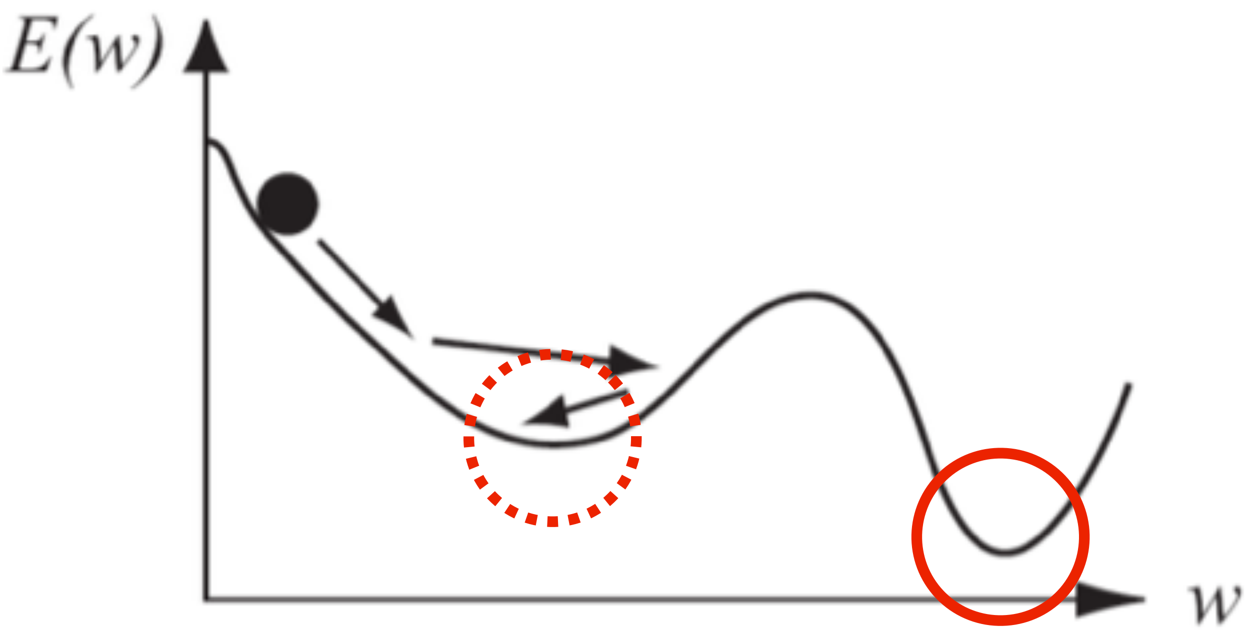

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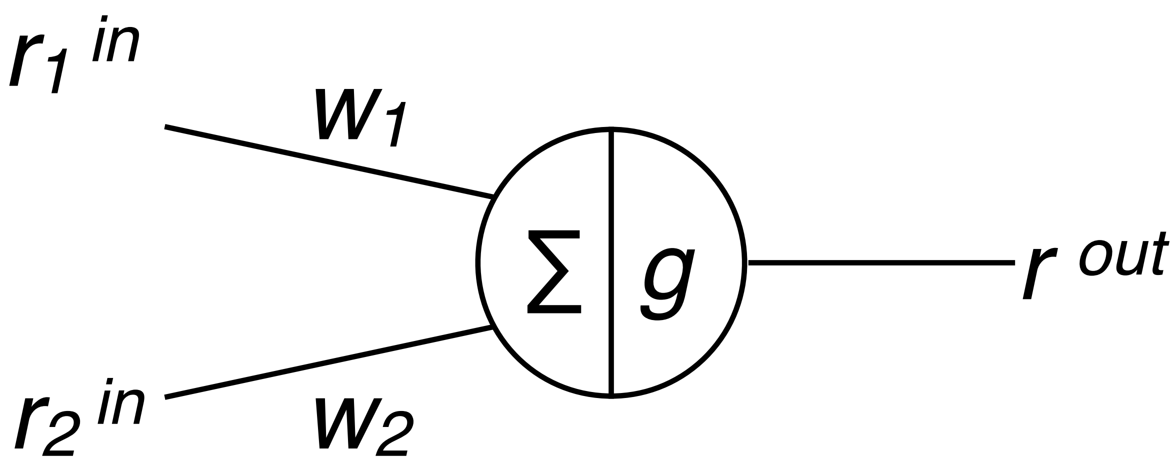
learning
rate

How do we train a neural network?

Minimize some cost function… gradient descent!



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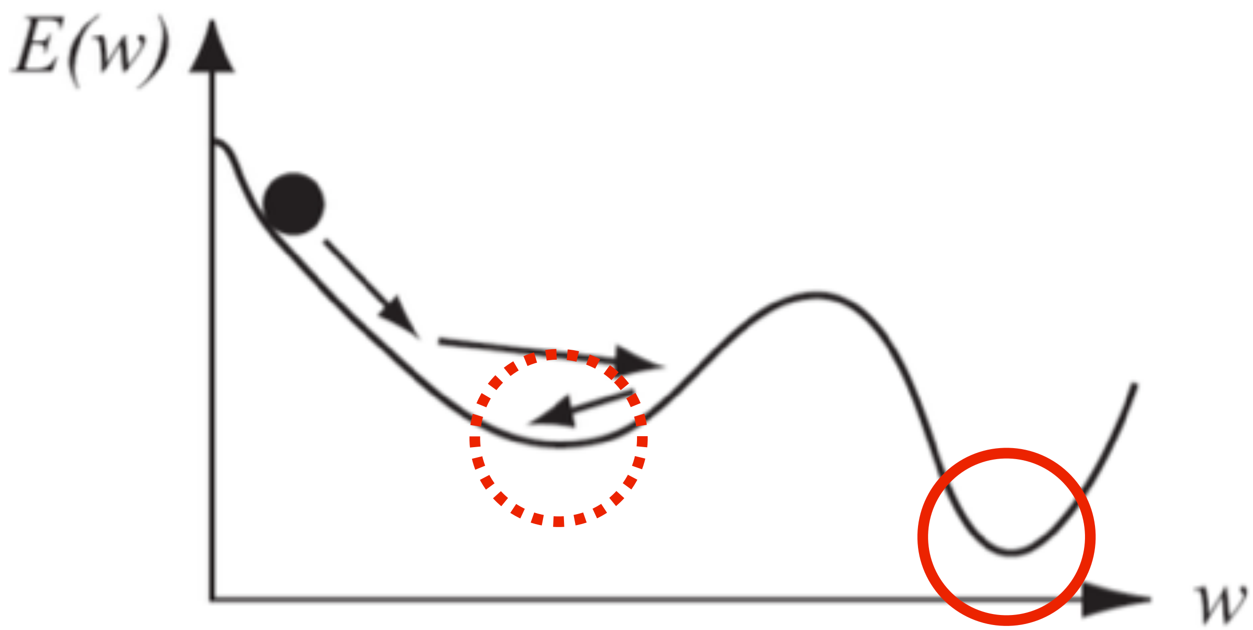

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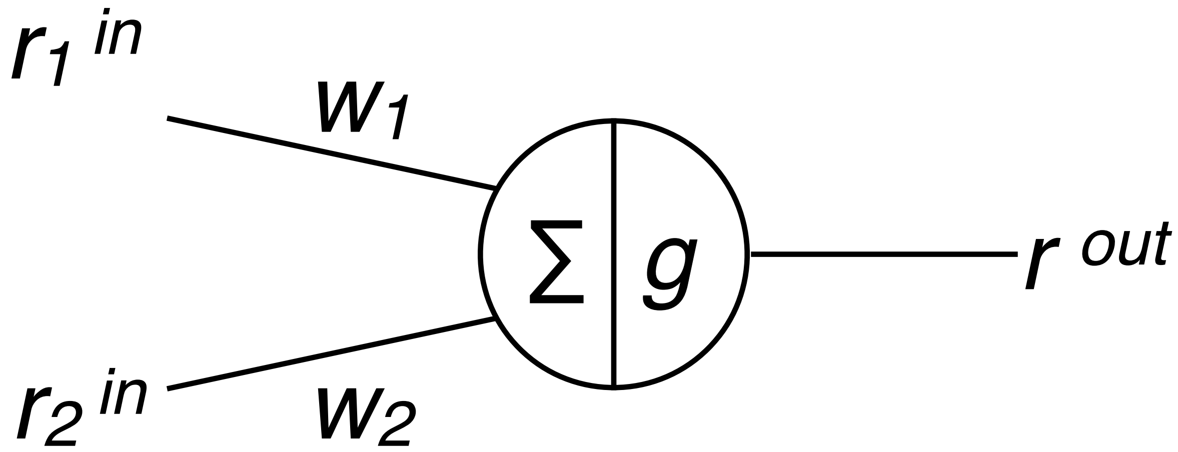
learning rate

gradient of MSE wrt weights

How do we train a neural network?



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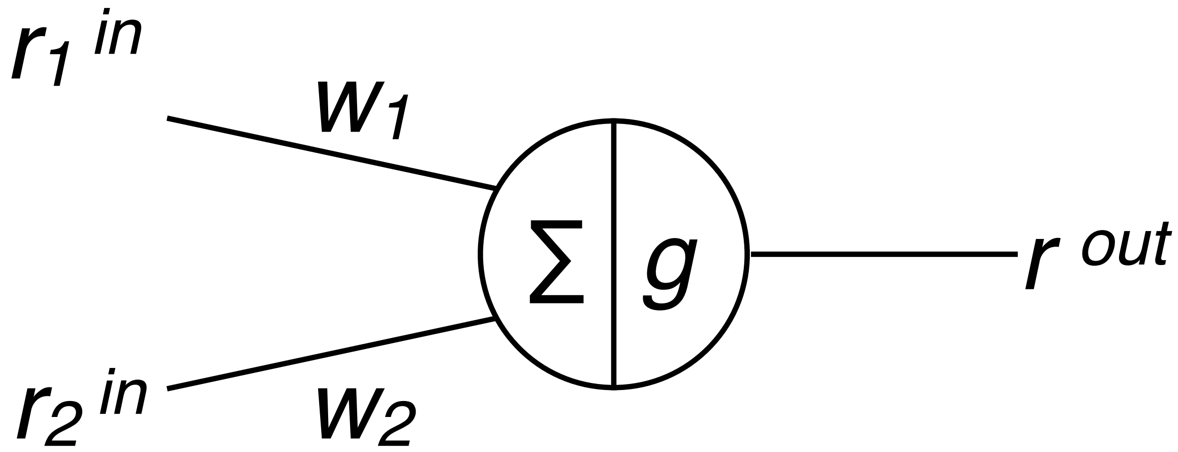

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change in weight i,j depends
on learning rate and dependence of
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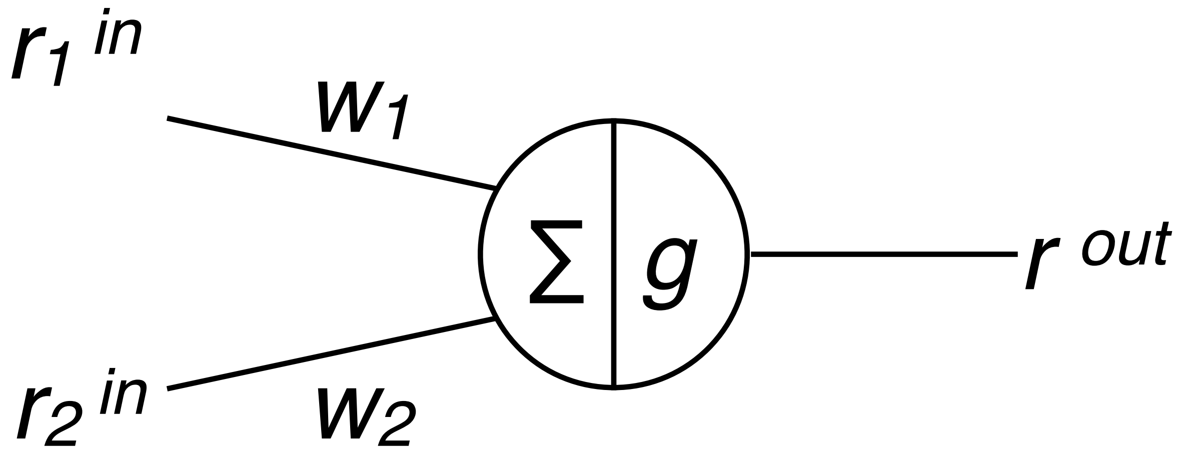

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$$\frac{\partial E}{\partial w_{ij}} = \frac{1}{2}\frac{\partial}{\partial w_{ij}} \underbrace{\sum_i \left(g\left(\underbrace{\sum_j w_{ij} r_j^{in}}_{h_i}\right) - y_i\right)^2}_{f}$$

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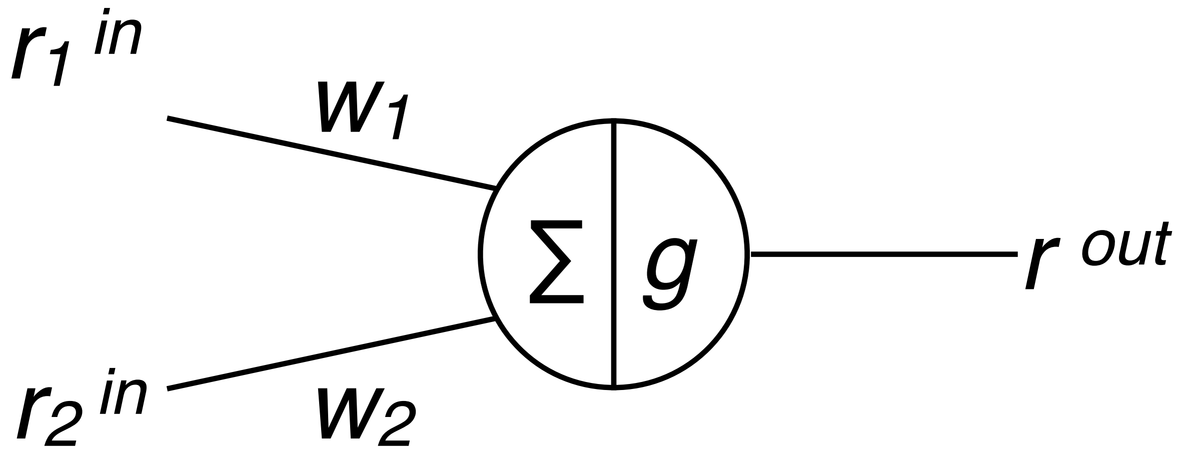

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error's dependence on weight i,j, rewritten using MSE equation

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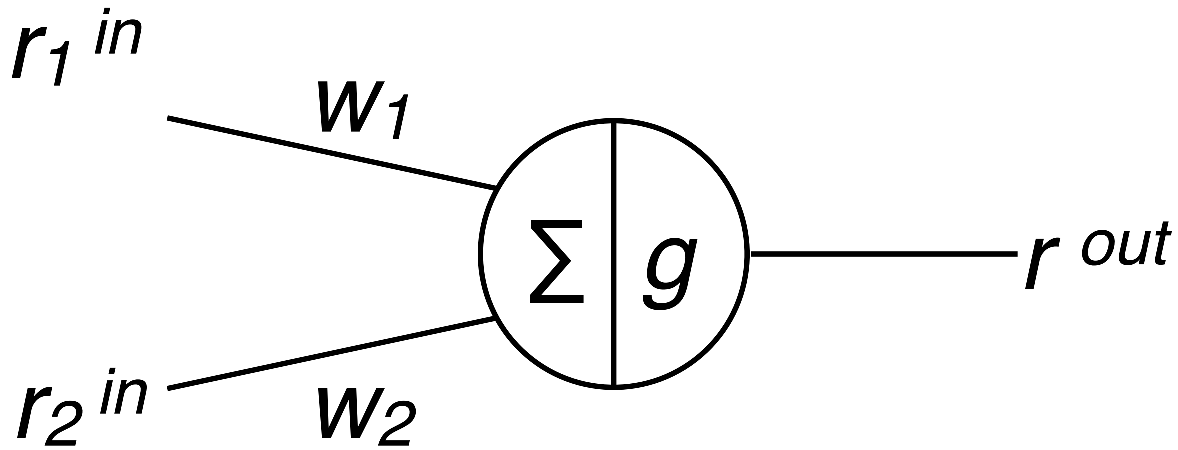

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error's dependence on weight i,j, rewritten
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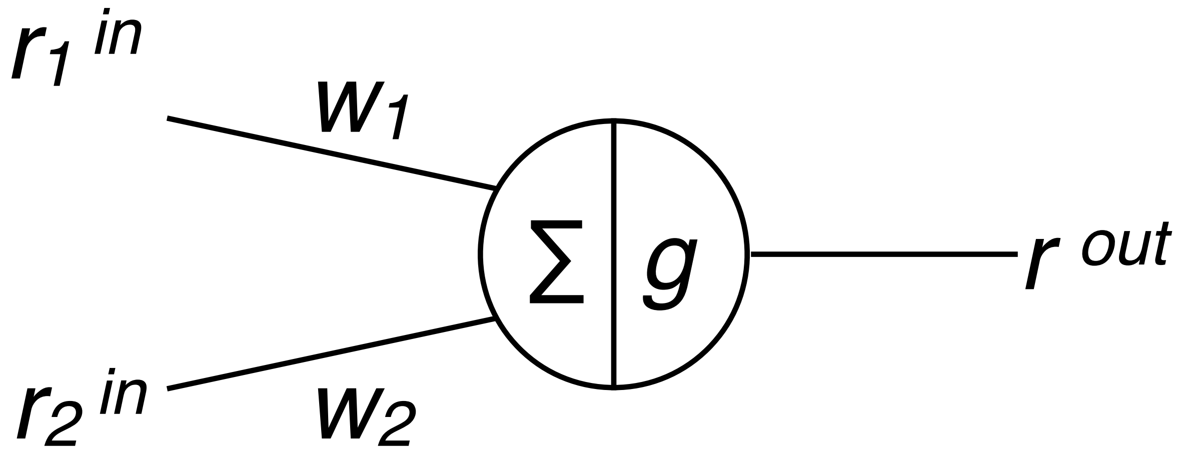
$$\frac{\partial f}{\partial w_{ij}} = \frac{\partial f}{\partial g}\frac{\partial g}{\partial h}\frac{\partial h}{\partial w_{ij}}$$

(using chain rule)

How do we train a neural network?



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$$\frac{\partial f}{\partial w_{ij}} = \frac{\partial f}{\partial g}\frac{\partial g}{\partial h}\frac{\partial h}{\partial w_{ij}}$$

(using chain rule)

$$\Delta w_{ij} = \varepsilon \left(g'(h_i)\left(y_i - r_i^{out}\right) r_j^{in}\right)$$ **learning rule** (derivation?)

Adding layers…

Start by finding the output rates…

2-layer (1 hidden) perceptron

$$E = \frac{1}{2}\sum_i \left(r_i^{out} - y_i\right)^2$$

$$\Delta w_{ij} = \varepsilon\left(g'\left(h_i\right)\left(y_i - r_i^{out}\right) r_j^{in}\right)$$

Adding layers…

Start by finding the output rates…

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$$\mathbf{r}^{out} = g\left(\mathbf{w}^{out}\mathbf{r}^{h}\right)$$

$$r_i^{out} = g\left(\sum_j w_{ij}^{out} r_j^{h}\right)$$

$$E = \frac{1}{2}\sum_i \left(r_i^{out} - y_i\right)^2$$

$$\Delta w_{ij} = \varepsilon\left(g'\left(h_i\right)\left(y_i - r_i^{out}\right) r_j^{in}\right)$$

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$$\mathbf{r}^{out} = g^{out}\left(\mathbf{w}^{out} g^{h}\left(\mathbf{w}^{h}\mathbf{r}^{in}\right)\right)$$

$$E = \frac{1}{2}\sum_i \left(r_i^{out} - y_i\right)^2$$

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Start by finding the output rates…

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$$\mathbf{r}^{out} = g^{out}\left(\mathbf{w}^{out} g^{h}\left(\mathbf{w}^{h}\mathbf{r}^{in}\right)\right)$$

3-layer (2 hidden) perceptron

$$\mathbf{r}^{out} = g^{out}\left(\mathbf{w}^{out} g^{h_{out-1}}\left(\mathbf{w}^{h_{out-1}} g^{h_{out-2}}\left(\mathbf{w}^{h_{out-2}}\mathbf{r}^{in}\right)\right)\right)$$

Adding layers…

Start by finding the output rates…

$$E = \frac{1}{2}\sum_i \left(r_i^{out} - y_i\right)^2$$

$$\Delta w_{ij} = \varepsilon \left(g'\left(h_i\right)\left(y_i - r_i^{out}\right) r_j^{in}\right)$$

2-layer (1 hidden) perceptron

$$\mathbf{r}^{out} = g\left(\mathbf{w}^{out}\mathbf{r}^h\right)$$

$$r_i^{out} = g\left(\sum_j w_{ij}^{out} r_j^h\right)$$

$$\mathbf{r}^{out} = g^{out}\left(\mathbf{w}^{out} g^h\left(\mathbf{w}^h \mathbf{r}^{in}\right)\right)$$

3-layer (2 hidden) perceptron

$$\mathbf{r}^{out} = g^{out}\left(\mathbf{w}^{out} g^{h_{out-1}}\left(\mathbf{w}^{h_{out-1}} g^{h_{out-2}}\left(\mathbf{w}^{h_{out-2}}\mathbf{r}^{in}\right)\right)\right)$$

n-layer (n-1 hidden) perceptron

$$\mathbf{r}^{out} = g^{out}\left(\mathbf{w}^{out} g^{h_{out-1}}\left(\mathbf{w}^{h_{out-1}} g^{h_{out-2}}\left(\mathbf{w}^{h_{out-2}} \ldots g^{h_{out-n+1}}\left(\mathbf{w}^{h_{out-n+1}} g^{h_{out-n}}\left(\mathbf{w}^{h_{out-n}}\mathbf{r}^{in}\right)\right)\right)\right)\right)$$

Adding layers…

Start by finding the output rates…

2-layer (1 hidden) perceptron

$$\mathbf{r}^{out} = g\left(\mathbf{w}^{out}\mathbf{r}^{h}\right)$$

$$r_i^{out} = g\left(\sum_j w_{ij}^{out} r_j^{h}\right)$$

$$E = \frac{1}{2}\sum_i \left(r_i^{out} - y_i\right)^2$$

$$\Delta w_{ij} = \varepsilon\left(g'\left(h_i\right)\left(y_i - r_i^{out}\right) r_j^{in}\right)$$

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Idea is to nest each layer's output rates within the next…

Training through the layers…

Generalized delta rule (output wts)

$$\frac{\partial E}{\partial w_{ij}^{out}} = \frac{1}{2}\frac{\partial}{\partial w_{ij}^{out}}\sum_i \left(r_i^{out} - y_i\right)^2$$

$$E = \frac{1}{2}\sum_i \left(r_i^{out} - y_i\right)^2$$

$$\Delta w_{ij} = \varepsilon \left(g'\left(h_i\right)\left(y_i - r_i^{out}\right) r_j^{in}\right)$$

Training through the layers…

Generalized delta rule (output wts)

$$\frac{\partial E}{\partial w_{ij}^{out}} = \frac{1}{2}\frac{\partial}{\partial w_{ij}^{out}}\sum_i \left(r_i^{out} - y_i\right)^2$$
$$= \delta_i^{out} r_j^h$$

$$E = \frac{1}{2}\sum_i \left(r_i^{out} - y_i\right)^2$$

$$\Delta w_{ij} = \varepsilon \left(g'\left(h_i\right)\left(y_i - r_i^{out}\right) r_j^{in}\right)$$

Training through the layers…

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$$= \delta_i^{out} r_j^h$$

with

$$\delta_i^{out} = g^{out}{}'\left(h_i^h\right)\left(r_i^{out} - y_i\right)$$ **delta rule for output weights**

Training through the layers…

$$E = \frac{1}{2}\sum_i \left(r_i^{out} - y_i\right)^2$$

$$\Delta w_{ij} = \varepsilon \left(g'\left(h_i\right)\left(y_i - r_i^{out}\right) r_j^{in}\right)$$

Generalized delta rule (output wts)

$$\frac{\partial E}{\partial w_{ij}^{out}} = \frac{1}{2}\frac{\partial}{\partial w_{ij}^{out}}\sum_i \left(r_i^{out} - y_i\right)^2$$
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with

$$\delta_i^{out} = g^{out\prime}\left(h_i^h\right)\left(r_i^{out} - y_i\right)$$ **delta rule for output weights**

Hidden layer weights

Training through the layers…

$$E = \frac{1}{2}\sum_i \left(r_i^{out} - y_i\right)^2$$

$$\Delta w_{ij} = \varepsilon\left(g'\left(h_i\right)\left(y_i - r_i^{out}\right) r_j^{in}\right)$$

Generalized delta rule (output wts)

$$\frac{\partial E}{\partial w_{ij}^{out}} = \frac{1}{2}\frac{\partial}{\partial w_{ij}^{out}}\sum_i \left(r_i^{out} - y_i\right)^2$$
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with

$$\delta_i^{out} = g^{out}{}'\left(h_i^h\right)\left(r_i^{out} - y_i\right)$$ **delta rule for output weights**

Hidden layer weights

$$\frac{\partial E}{\partial w_{ij}^{h}} = \frac{1}{2}\frac{\partial}{\partial w_{ij}^{h}}\sum_i \left(r_i^{out} - y_i\right)^2$$

Training through the layers…

$$E = \frac{1}{2}\sum_i \left(r_i^{out} - y_i\right)^2$$

$$\Delta w_{ij} = \varepsilon\left(g'\left(h_i\right)\left(y_i - r_i^{out}\right) r_j^{in}\right)$$

Generalized delta rule (output wts)

$$\frac{\partial E}{\partial w_{ij}^{out}} = \frac{1}{2}\frac{\partial}{\partial w_{ij}^{out}}\sum_i \left(r_i^{out} - y_i\right)^2$$
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with

$$\delta_i^{out} = g^{out\,\prime}\left(h_i^h\right)\left(r_i^{out} - y_i\right)$$ **delta rule for output weights**

Hidden layer weights

$$\frac{\partial E}{\partial w_{ij}^{h}} = \frac{1}{2}\frac{\partial}{\partial w_{ij}^{h}}\sum_i \left(r_i^{out} - y_i\right)^2$$

$$\frac{\partial E}{\partial w_{ij}^{h}} = \frac{1}{2}\frac{\partial}{\partial w_{ij}^{h}}\sum_i \left(g^{out}\left(\sum_j w_{ij}^{out} g^h\left(\sum_k w_{jk}^h r_k^{in}\right)\right) - y_i\right)^2$$
$$= \delta_i^h r_j^{in}$$

Training through the layers…

Generalized delta rule (output wts)

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with

$$\delta_i^{out} = g^{out\,\prime}\left(h_i^h\right)\left(r_i^{out} - y_i\right)$$

delta rule for output weights

Hidden layer weights

$$\frac{\partial E}{\partial w_{ij}^{h}} = \frac{1}{2}\frac{\partial}{\partial w_{ij}^{h}}\sum_i \left(r_i^{out} - y_i\right)^2$$

$$\frac{\partial E}{\partial w_{ij}^{h}} = \frac{1}{2}\frac{\partial}{\partial w_{ij}^{h}}\sum_i \left(g^{out}\left(\sum_j w_{ij}^{out} g^h\left(\sum_k w_{jk}^h r_k^{in}\right)\right) - y_i\right)^2$$
$$= \delta_i^h r_j^{in}$$

with

$$\delta_i^h = g^{h\,\prime}\left(h_i^{in}\right)\sum_k w_{ik}^{out}\delta_k^{out}$$

delta rule for hidden wts
(depends on delta rule for output wts)

Training through the layers…

$$E = \frac{1}{2}\sum_i \left(r_i^{out} - y_i\right)^2$$

$$\Delta w_{ij} = \varepsilon \left(g'\left(h_i\right)\left(y_i - r_i^{out}\right) r_j^{in}\right)$$

Generalized delta rule (output wts)

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Error propagates BACKWARDS through the layers!

Hidden layer weights

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2. Test generalization of network using previously unseen test data set.

Caveats

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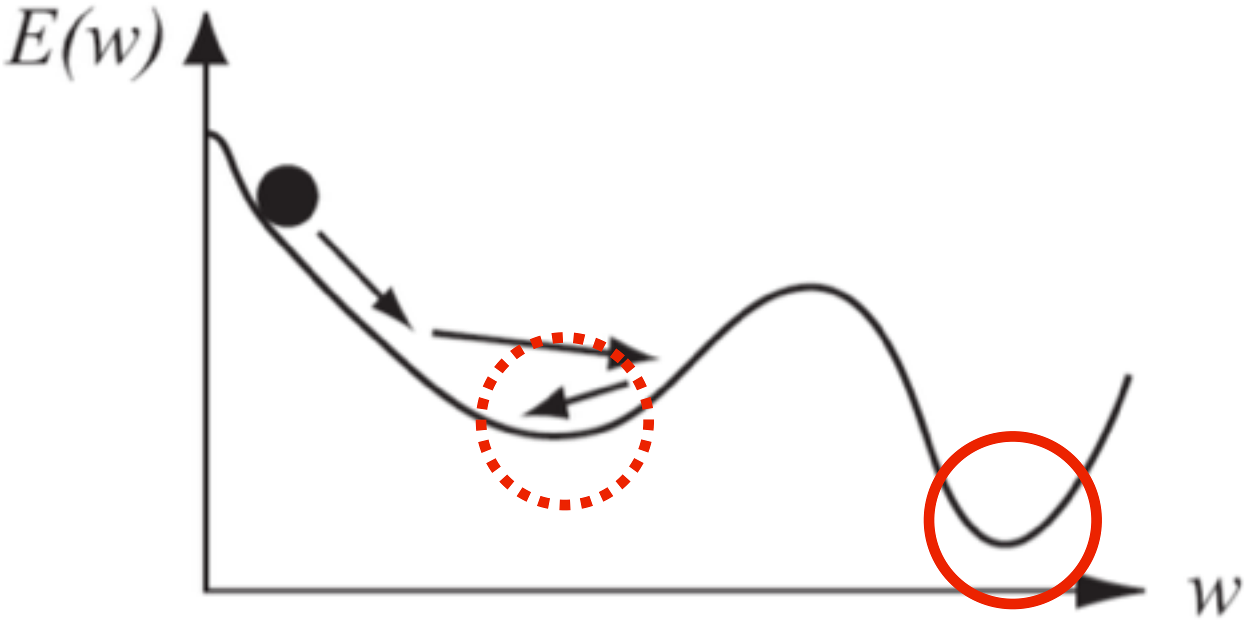

Local minima
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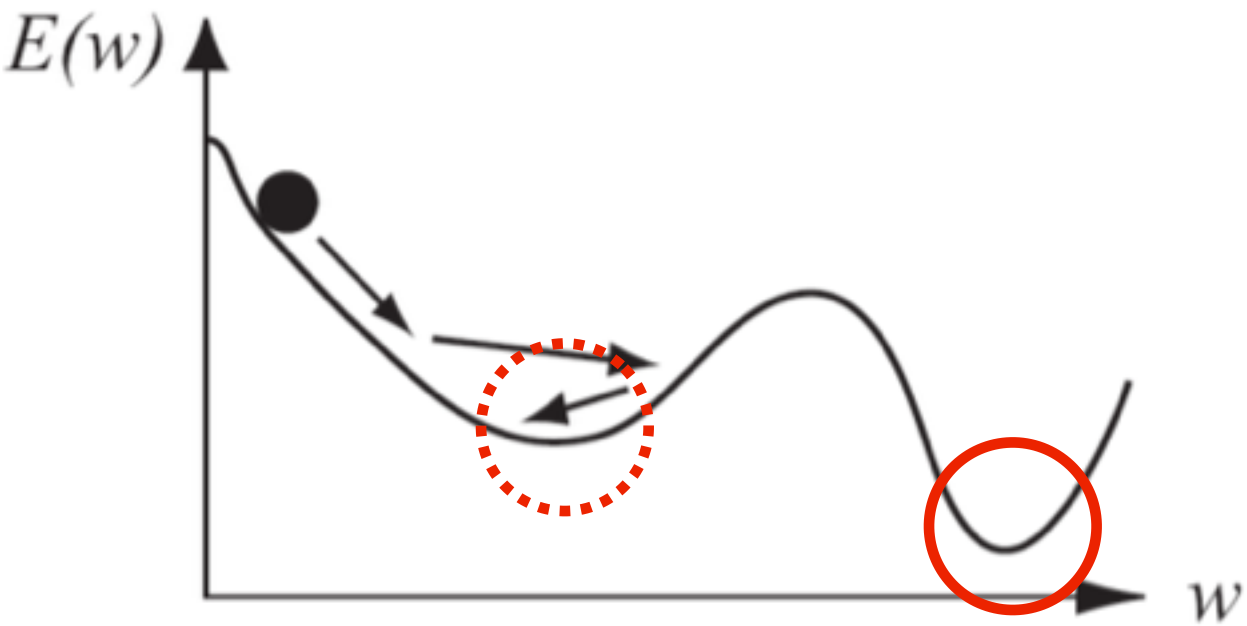

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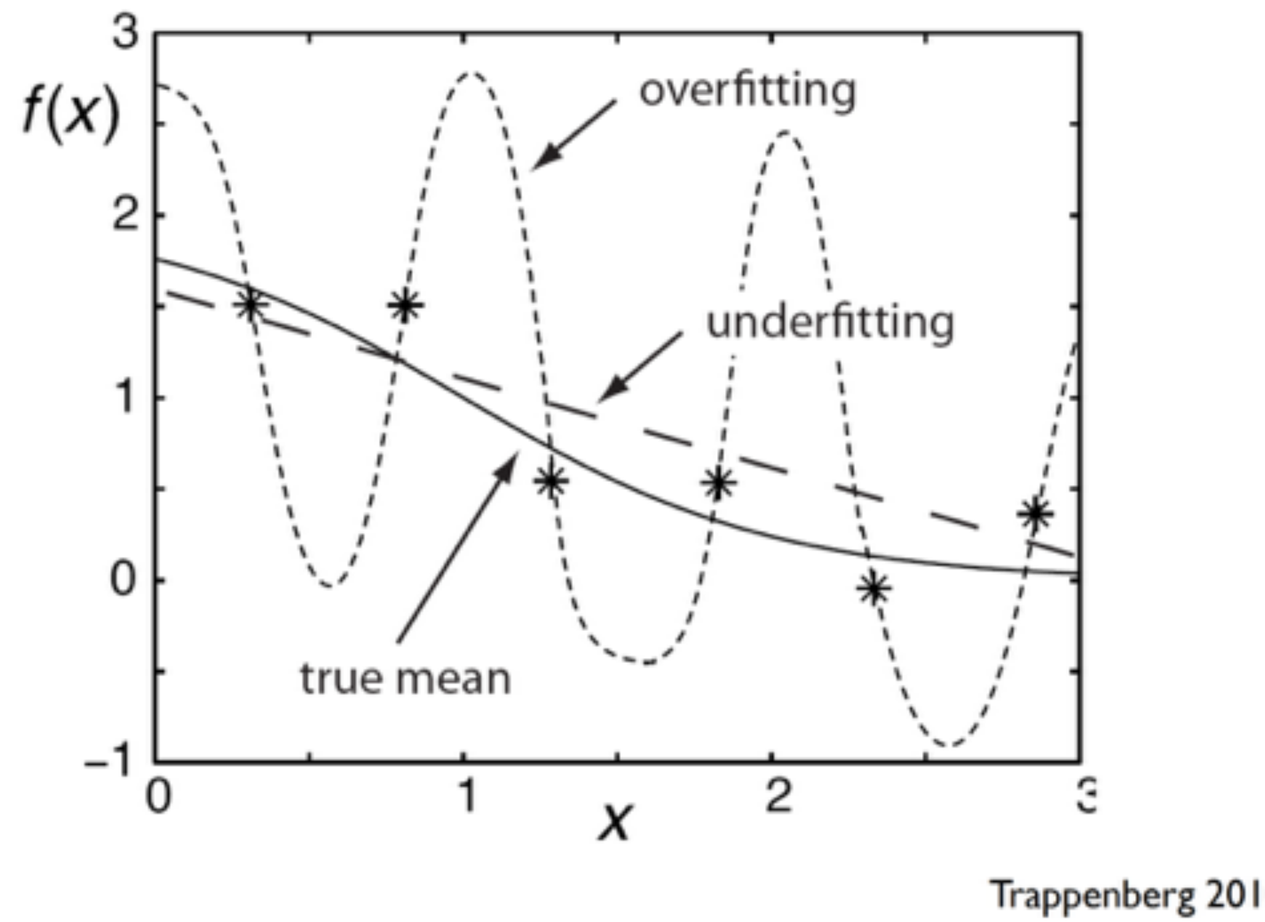

Overfitting
Use heuristics to determine appropriate number of nodes for solving particular problem. Can usually use 2*number of nodes for training set.

Recurrent networks are a whole new game!

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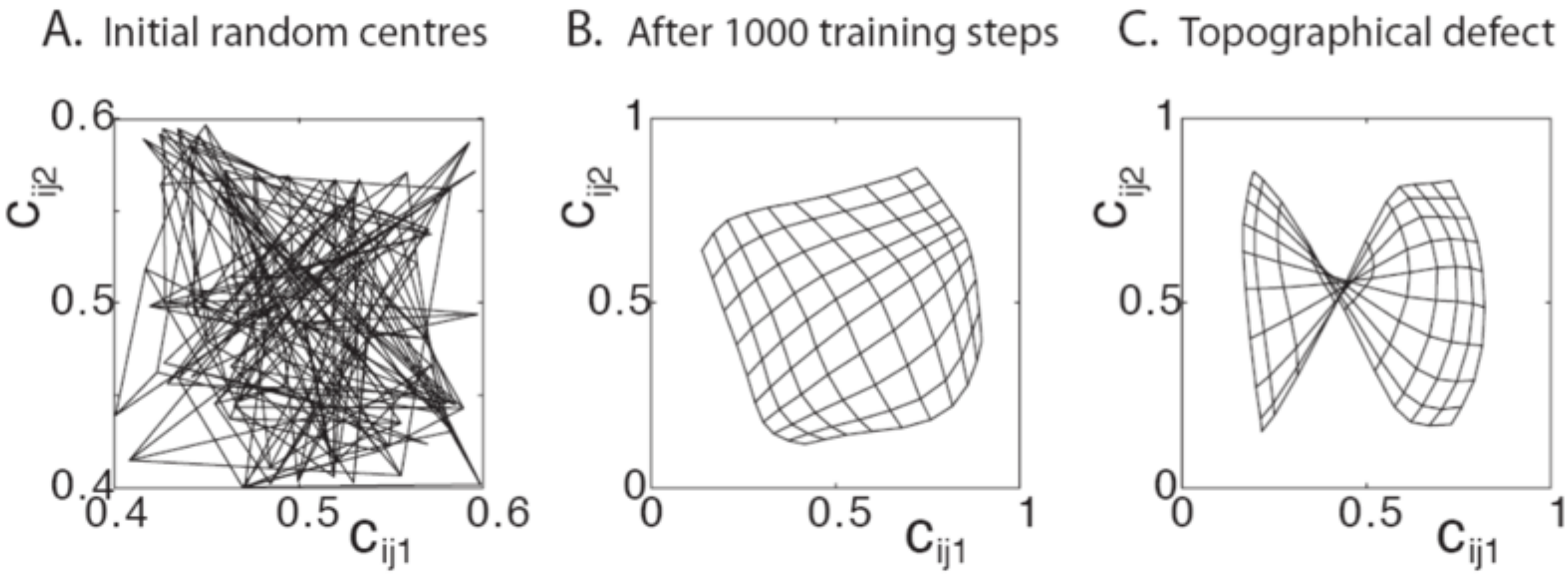
but I'll spare you



Trappenberg 2010

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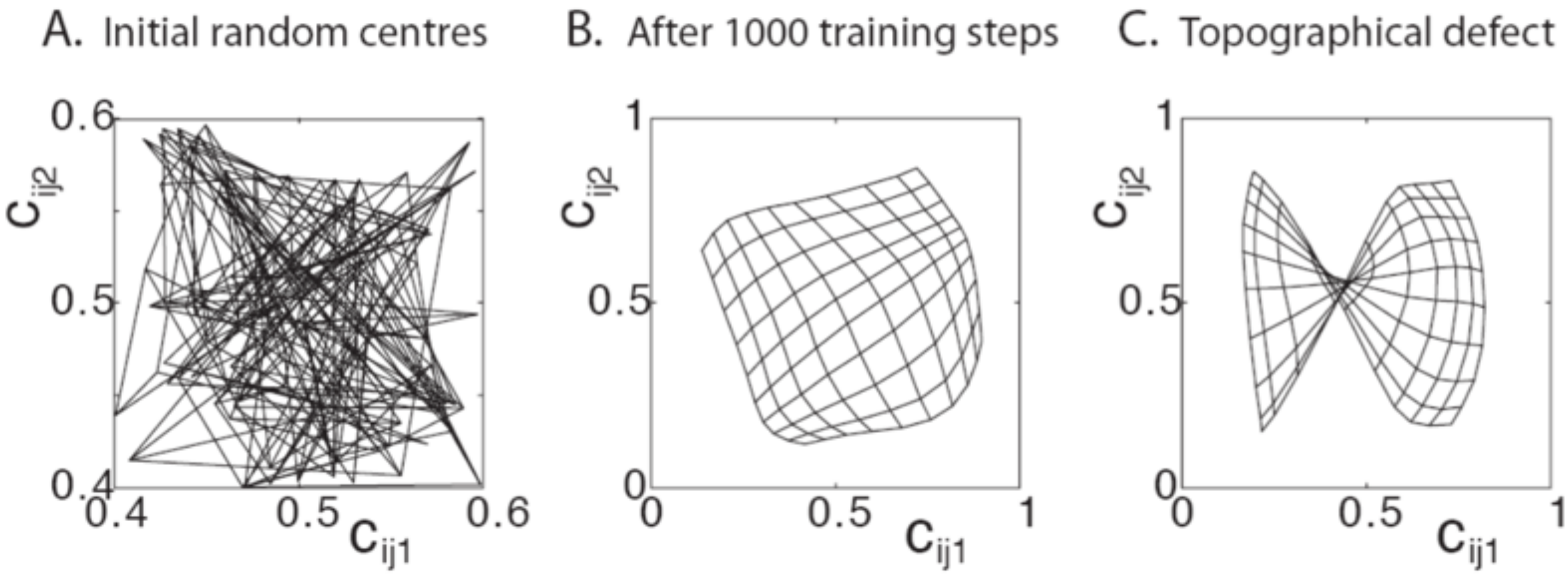

Trappenberg 2010

Backprop. through time (BPTT)