

A close-up photograph of a human eye. The eye is brown with a prominent starburst reflection on the iris. The eyelashes are dark and well-defined. The skin around the eye is light and shows some texture. The background is out of focus, showing another eye and part of the face.

# **Modeling eye movements**

**CoSMo 2017  
Gunnar Blohm**

# Introduction

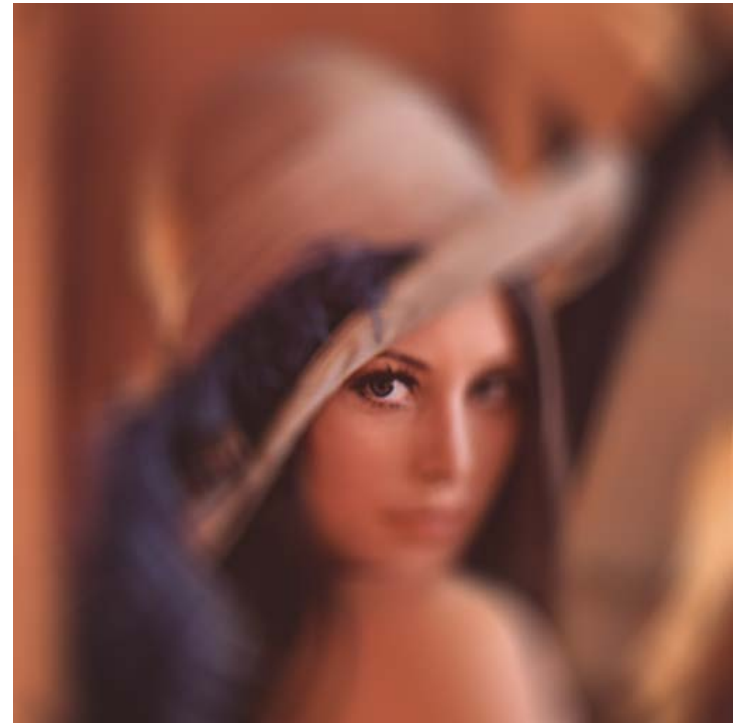
---

## ▶ Why move the eyes?

What we think we see



What we see



Lena, *Playboy* centerfold, Nov 1972  
(standard test image in the field of image processing since 1973)

# Introduction

---

- ▶ **Why move the eyes?**
  - ▶ **Functional role of eye movements: optimize visual processing!**
    - ▶ **Image stabilization**
      - Smooth pursuit of moving objects (low-pass)
      - Vestibulo-ocular reflex (VOR) compensates for head movements (high-pass)
      - Opto-kinetic nystagmus (OKN) stabilizes wide-field motion
    - ▶ **Image exploration**
      - Saccades re-orient gaze
  - ▶ **Resulting constraints for the controller(s)**
    - ▶ Minimize retinal error (position)
    - ▶ Minimize retinal image slip (velocity)
    - ▶ Be as fast and accurate as possible (role of predictions/anticipations...)

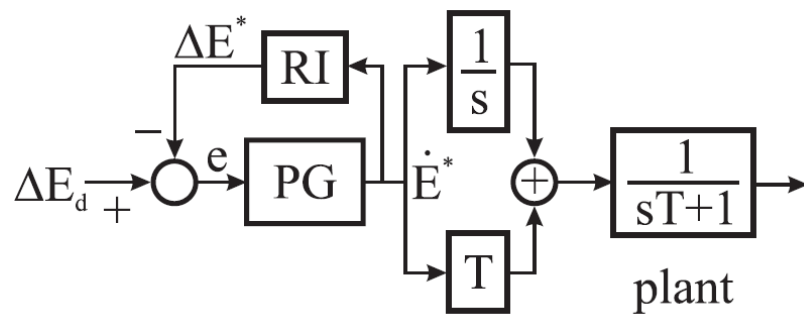


# Introduction

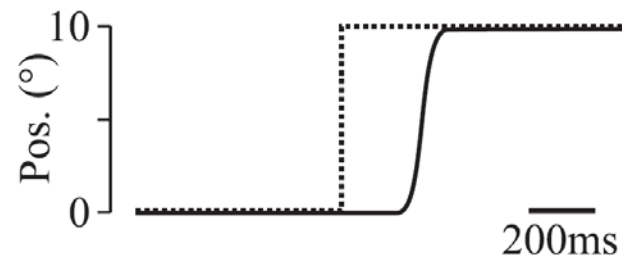
## ▶ Example: saccade model



Alfred Yarbus, 1967



Adapted from:  
Jürgens et al. 1981  
Scudder 1988



# Where to start?

---

- ▶ 2 potential approaches
  - ▶ Blind approach
    - ▶ Straight use of systems identification tools
    - ▶ Model the system based on the measured input-output relationship
    - ▶ = Engineering approach
  - ▶ Informed approach
    - ▶ Model systems components (not the system as a whole)
    - ▶ Incorporate detailed knowledge about physiology, biomechanics etc
    - ▶ = Neuroscience approach

# Eye/head plants

- ▶ Eye and neck muscles properties
- ▶ Damped spring-mass system equivalent

Equation of motion of eye ball:

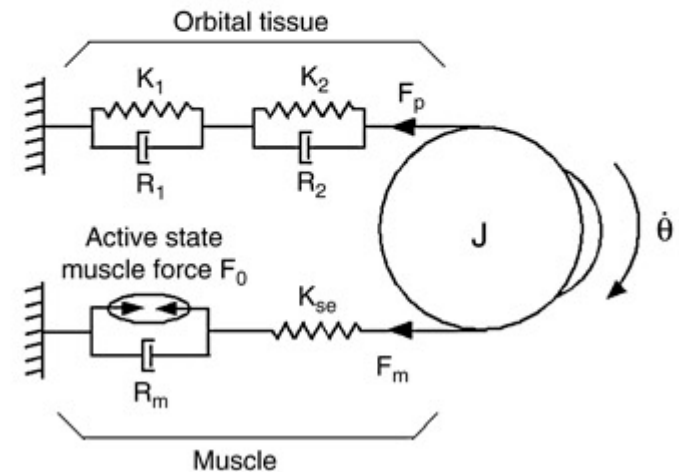
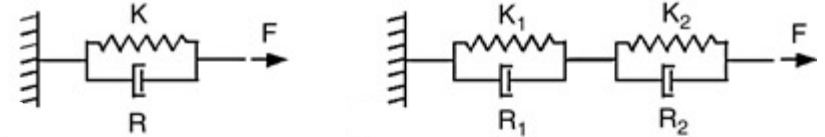
$$J \cdot \frac{d^2\theta}{dt^2} + F_p = F_m$$

Muscle force applied:

$$F_m = F_0 - R_m \cdot \frac{d\theta}{dt} - \frac{R_m}{K_{se}} \cdot \frac{dF_m}{dt}$$

Passive muscle/tissue force:

$$F_p = \frac{R_1 R_2 \cdot \frac{d^2\theta}{dt^2} + (R_1 K_2 + R_2 K_1) \cdot \frac{d\theta}{dt} + K_1 K_2 \cdot \theta - (R_1 + R_2) \cdot \frac{dF_p}{dt}}{(K_1 + K_2)}$$

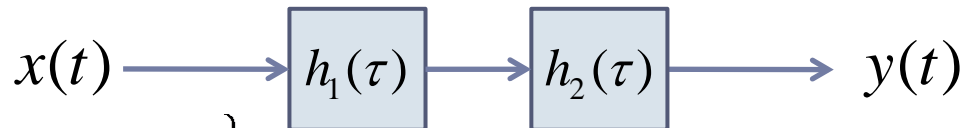


Robinson (1964), Scudder (2009)

J: moment of inertia  
 $F_p$ : passive force (muscle tissue)  
 $F_m$ : active muscle force

# Modelling the eye

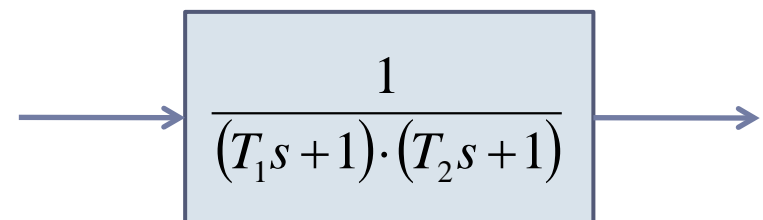
- ▶ Low inertia
  - ▶ Viscous muscles (like elastic bands)
- second-order linear system approximation



$$\left. \begin{aligned}
 u(t) &= \int_0^{\infty} x(t-\sigma) \cdot h_1(\sigma) d\sigma \\
 y(t) &= \int_0^{\infty} u(t-\tau) \cdot h_2(\tau) d\tau
 \end{aligned} \right\} \Rightarrow y(t) = \int_0^{\infty} \int_{\tau}^{\infty} x(t-\sigma-\tau) \cdot h_1(\sigma-\tau) \cdot h_2(\tau) d\sigma d\tau$$

(leads to terrible expressions!)

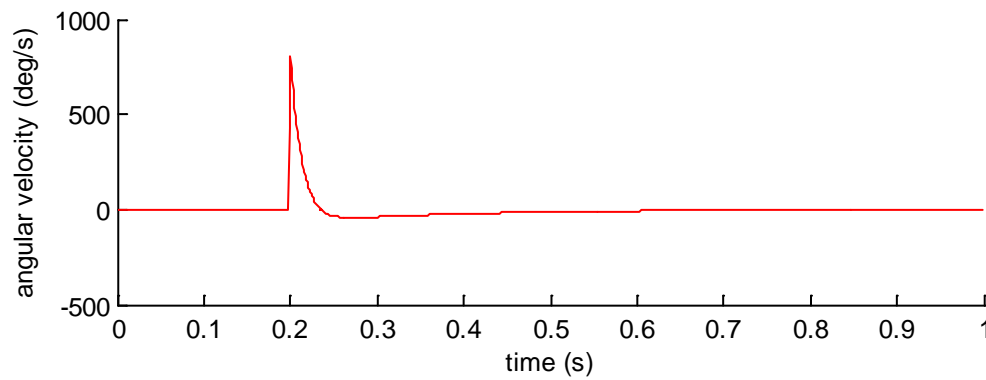
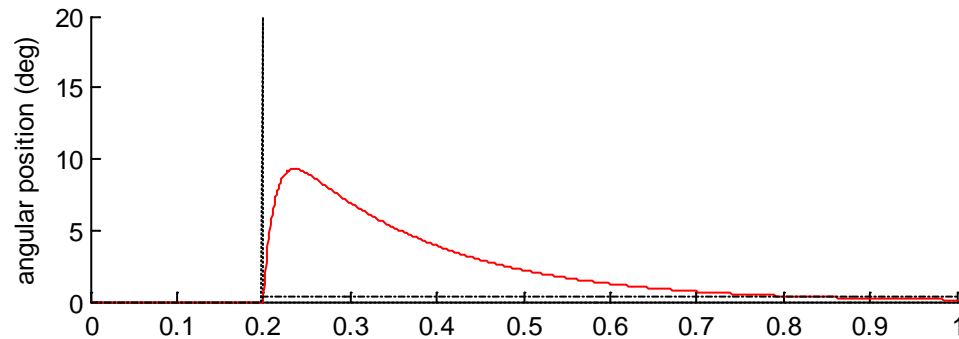
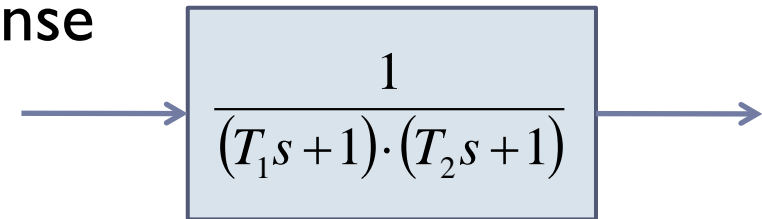
$$y(t) = h_1(t) * h_2(t) * x(t)$$



- ▶  $T_1 = 175\text{ms}, T_2 = 13\text{ms}$
- ▶  $T_1$ : viscosity of the plant
- ▶  $T_2$ : inertial properties of eye

# Modelling the eye

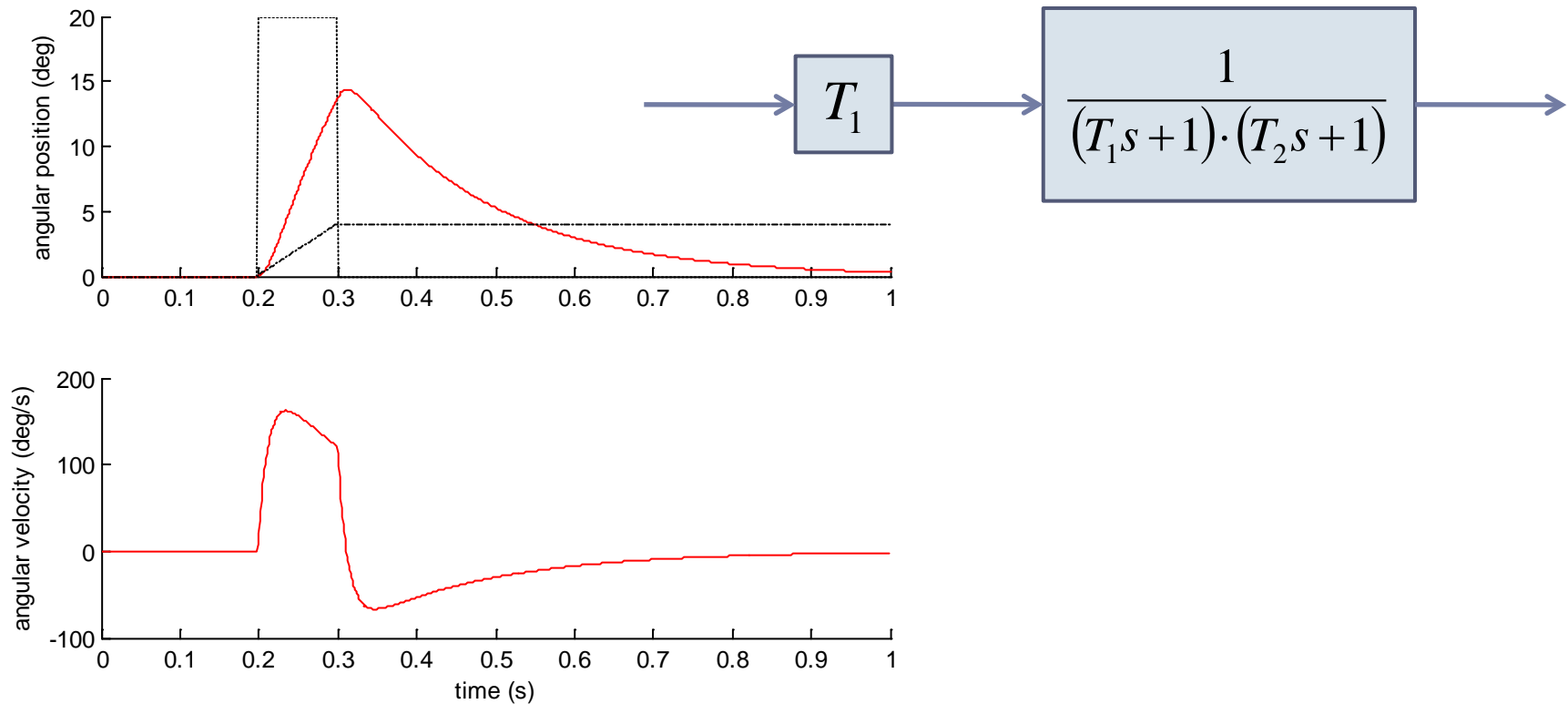
## ▶ Testing the model's impulse response





# Moving the eyes

- ▶ We need a motor command (phasic) to move the eyes
  - ▶ Eye moves, but gaze cannot be held

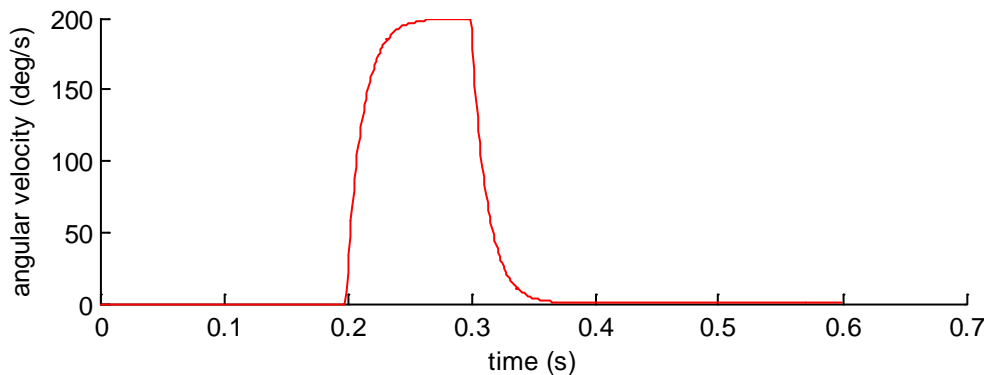
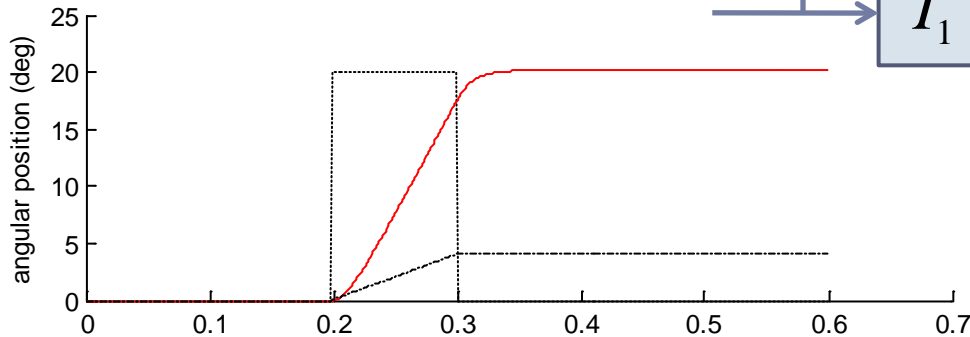
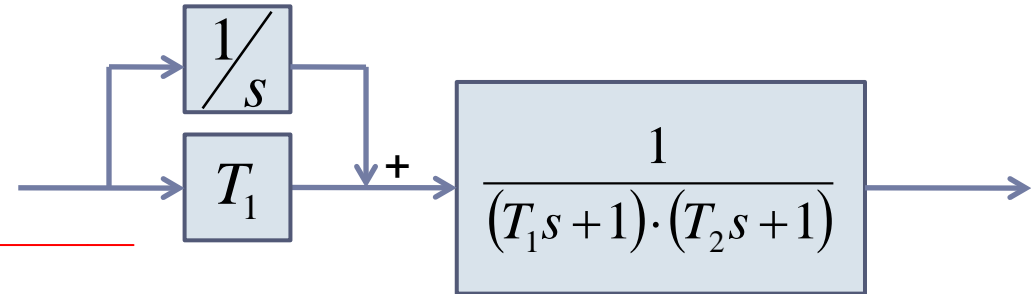


- ▶ A tonic activation needed to maintain gaze

# Gaze holding: the neural integrator

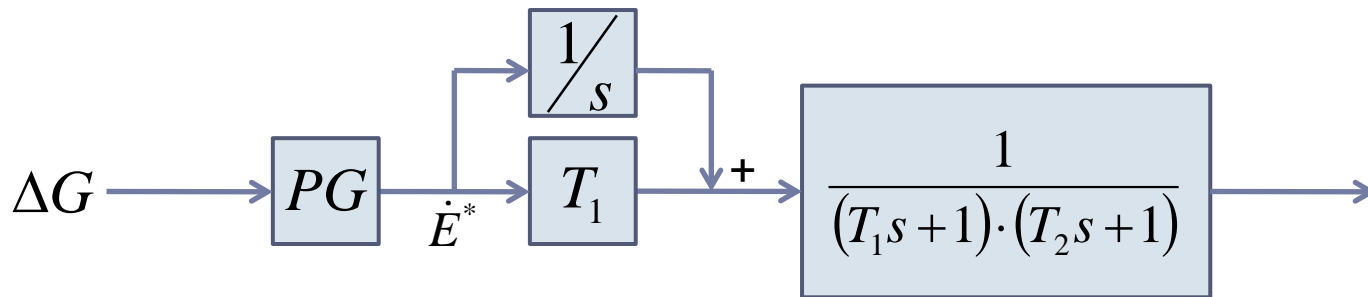
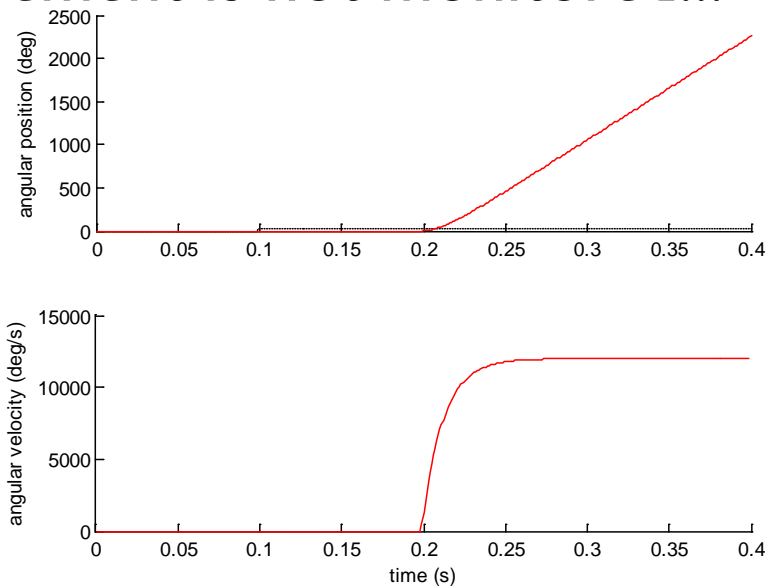
## ► How do we get the tonic portion?

### ► Neural integrator



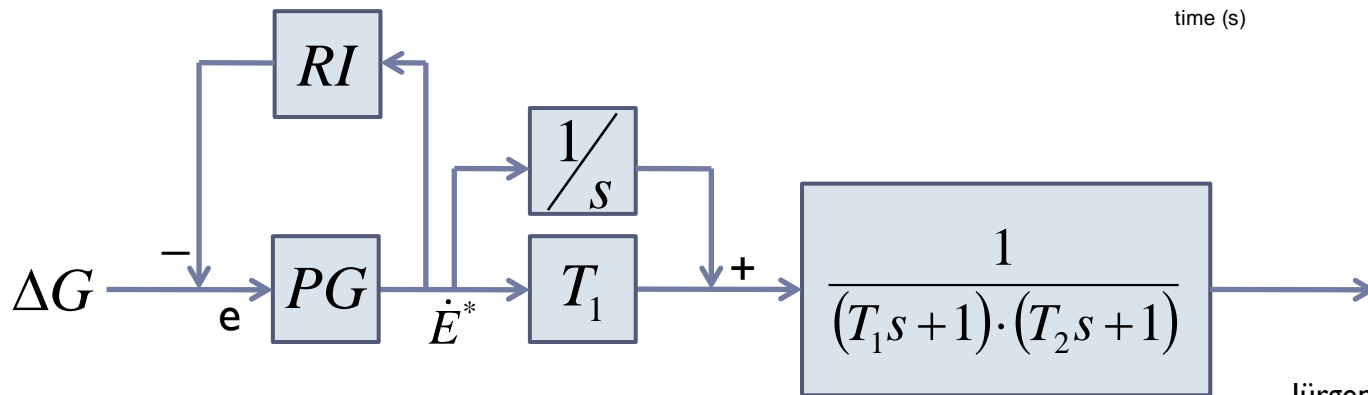
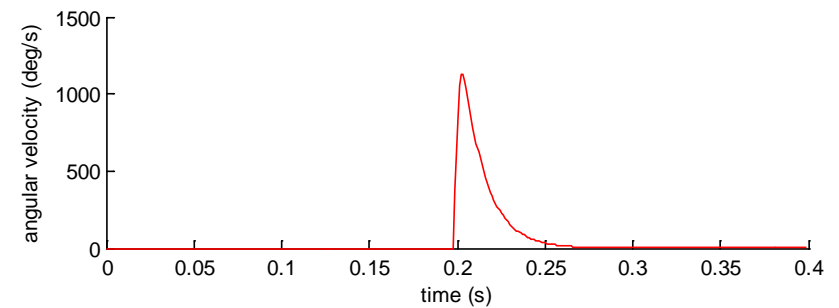
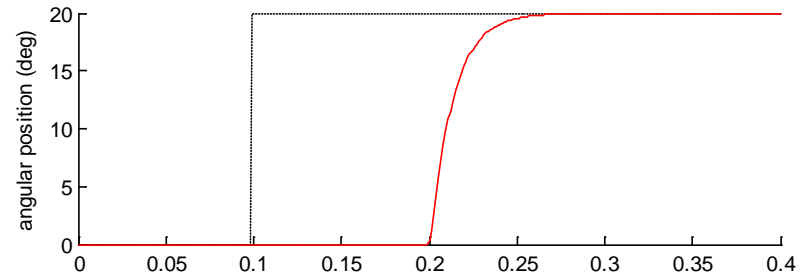
# Motor command generation

- ▶ Problem: movement is not monitored...



# Motor command generation

- ▶ **Solution: resettable integrator**
  - ▶ Only remaining error  $e$  drives the eyes



Jürgens et al. 1981  
Scudder 1988

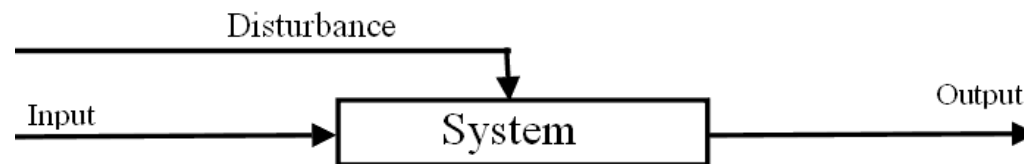
# Control modes

# Control modes

---

## ▶ Open-loop control models

- ▶ control only possible when system is well-defined, i.e. there is a precise mathematical relationship between input and output
- ▶ No knowledge of perturbations
- ▶ No learning possible
- ▶ Unstable!



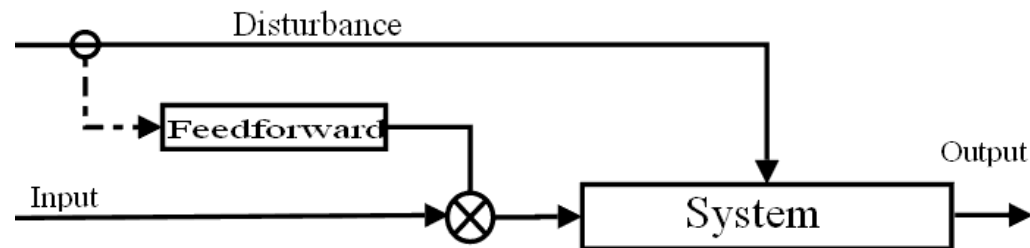
Wikipedia



# Control modes

---

- ▶ **Feed-forward control models (model predictive control)**
  - ▶ Uses an inverse model of the plant physics
  - ▶ Control output only depends on internal (inverse) model of the plant physics, independently of actual plant reaction
  - ▶ Feed-forward control without feedback = ballistic
    - ▶ Anticipatory regulation required for optimal behaviour
    - ▶ Based on learning
    - ▶ Neural network analogy / implementation: Perceptron



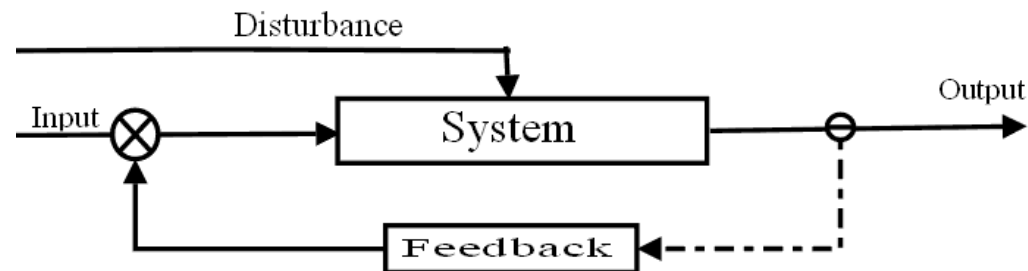
Wikipedia

# Control modes

---

## ▶ Feedback control models (closed-loop)

- ▶ Uses output history to adjust future commands
- ▶ Negative feedback
  - ▶ Helps maintaining stability in the system despite of external changes
  - ▶ Reduction of the input that caused the output
- ▶ Positive feedback
  - ▶ Amplifies the event that caused the output
  - ▶ Auto-excitation: self-enforcing loop



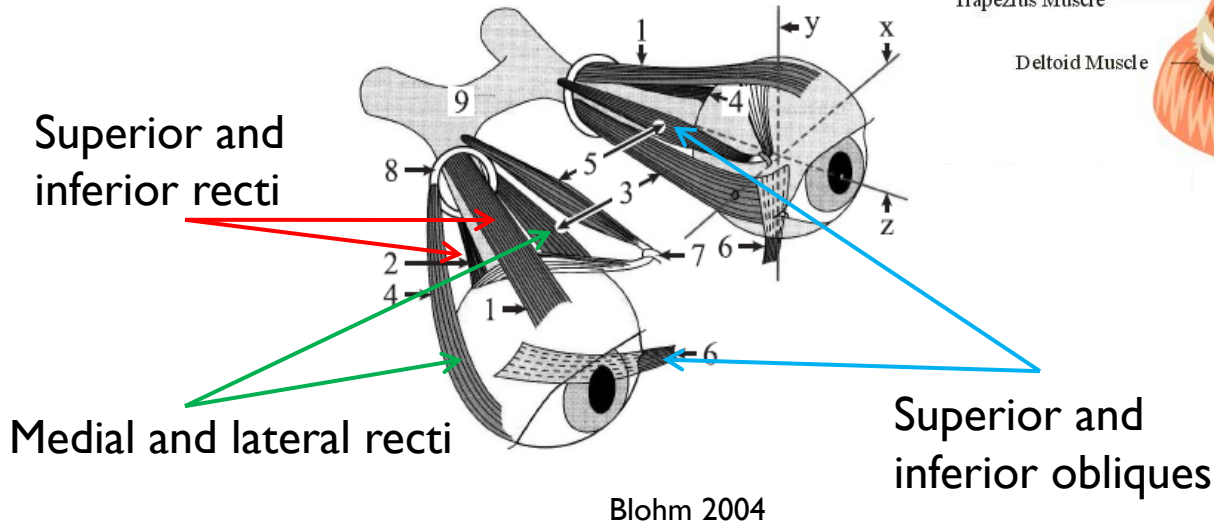
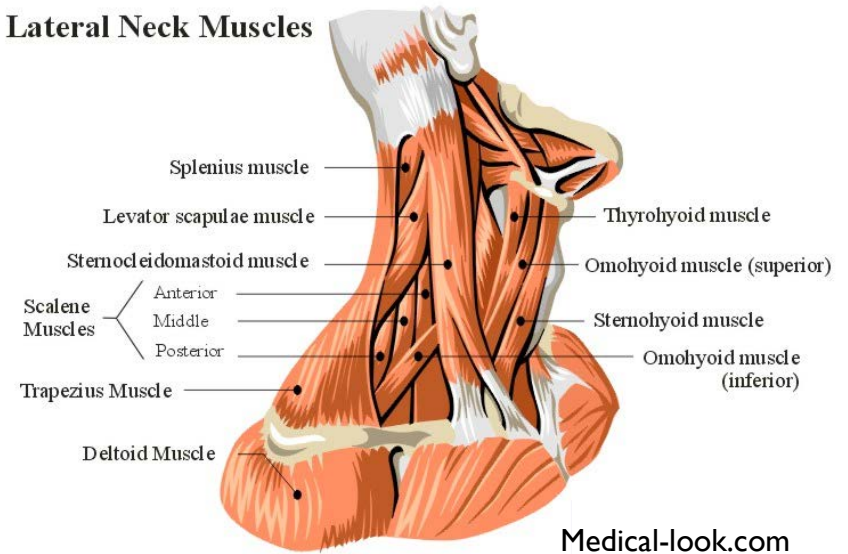
Wikipedia

# Eye movement models

# Eye movements

## ▶ Eye and neck muscles

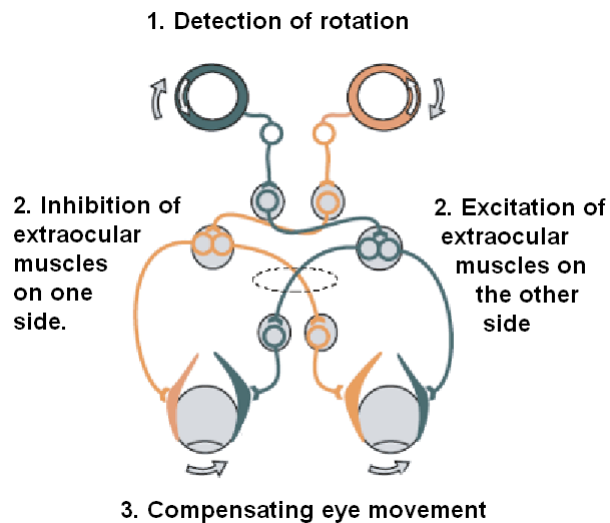
Lateral Neck Muscles



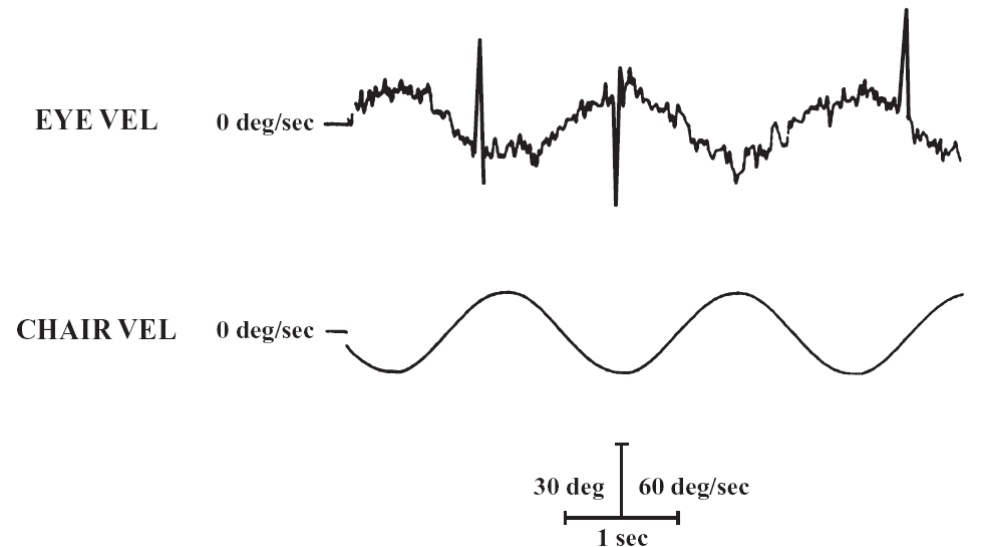
Blohm 2004

# Eye movements

## ▶ Vestibulo-ocular reflex



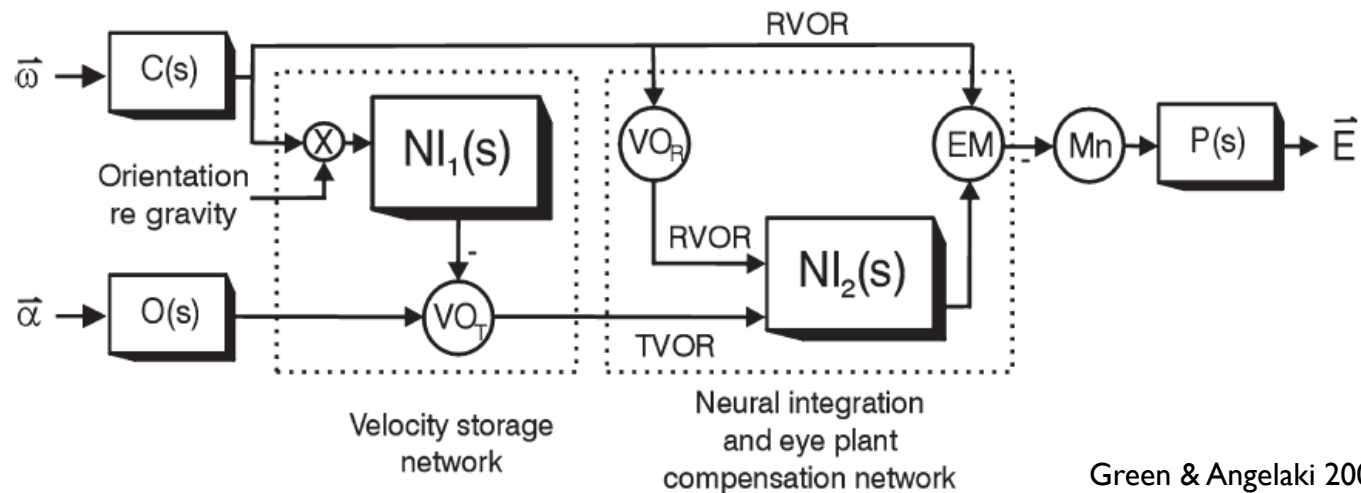
Wikipedia



Leigh & Zee 2006

# Eye movements

## ▶ Vestibulo-ocular reflex



- Angular velocity from semi-circular canals:  $\omega$
- Linear acceleration senses by otolith organs:  $\alpha$
- VO: vestibular-only neurons
- EM: eye movement sensitive neurons



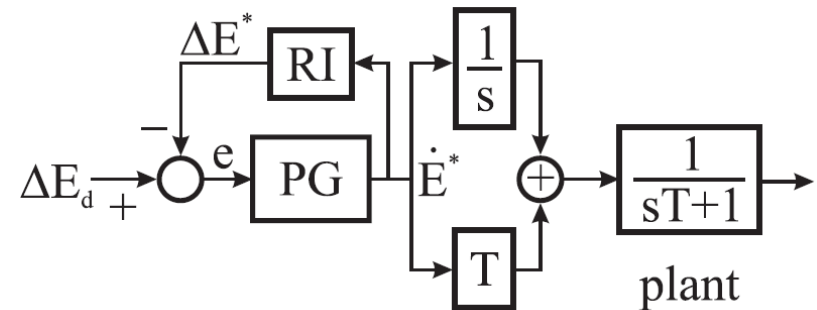


# Eye movements

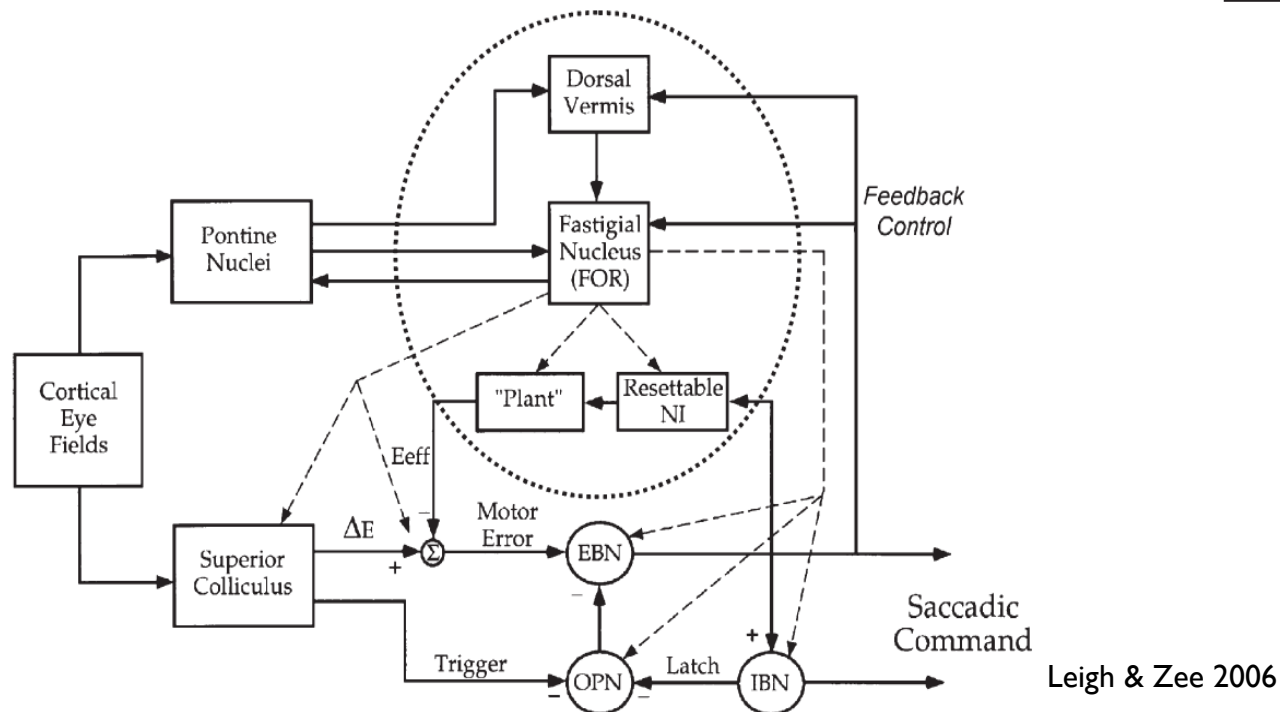
## ▶ Saccades:

### ▶ Executive control

- ▶ Ballistic: Robinson model
- ▶ Feedback control



Adapted from:  
Jürgens et al. 1981  
Scudder 1988

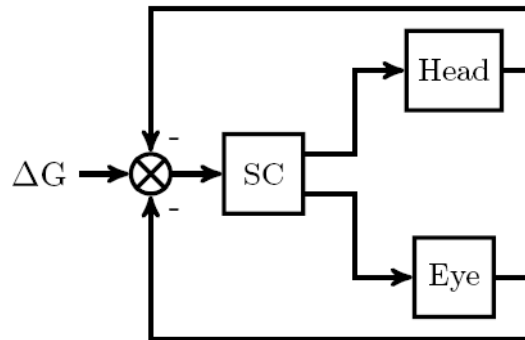


# Eye movements

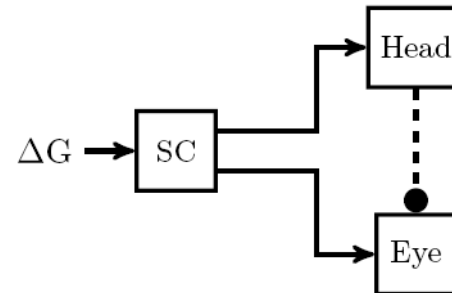
---

- ▶ Saccades:
  - ▶ Eye-head coordination: first models

Galiana, Lefèvre, Guitton (1992)



Freedman 2001, 2008

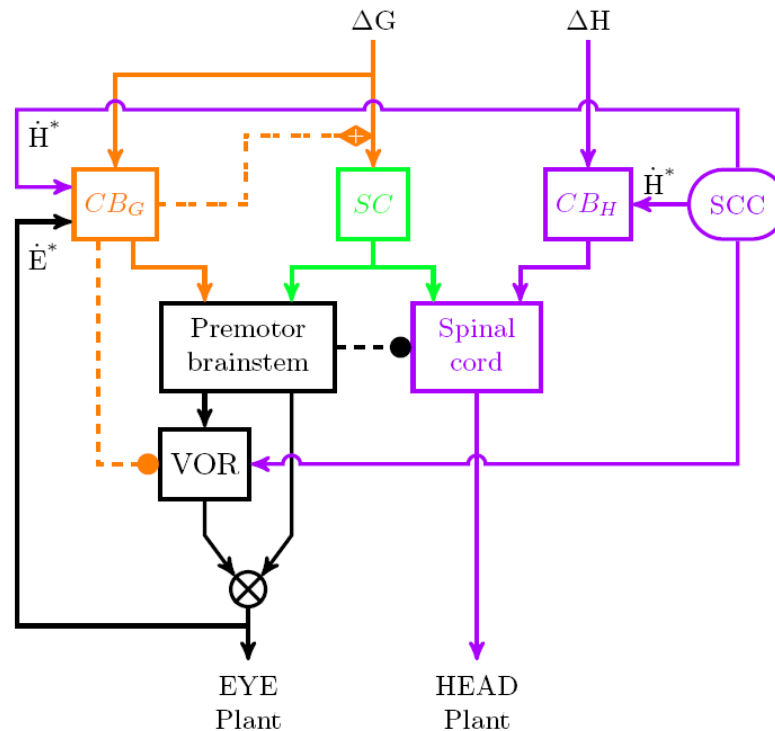


- ▶ Cannot model separate gaze and head goals

# Eye movements

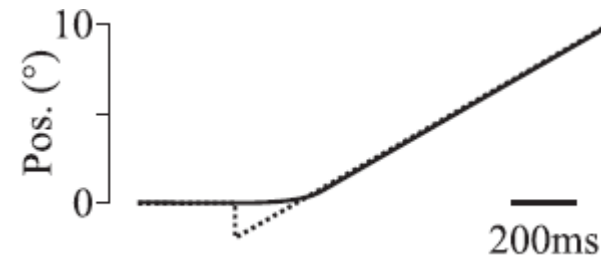
## ► Saccades:

- Eye-head coordination: new nested control model
  - Daye, Optican, Blohm, Lefèvre (2014)

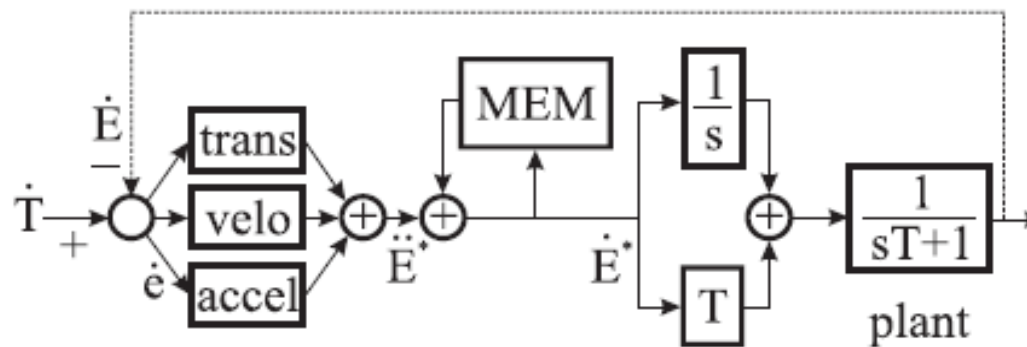


# Eye movements

- ▶ Smooth pursuit
  - ▶ Visually-guided pursuit



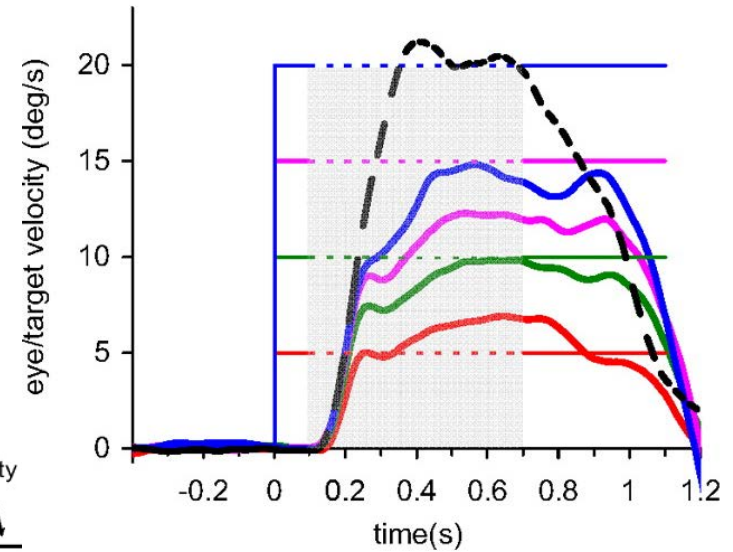
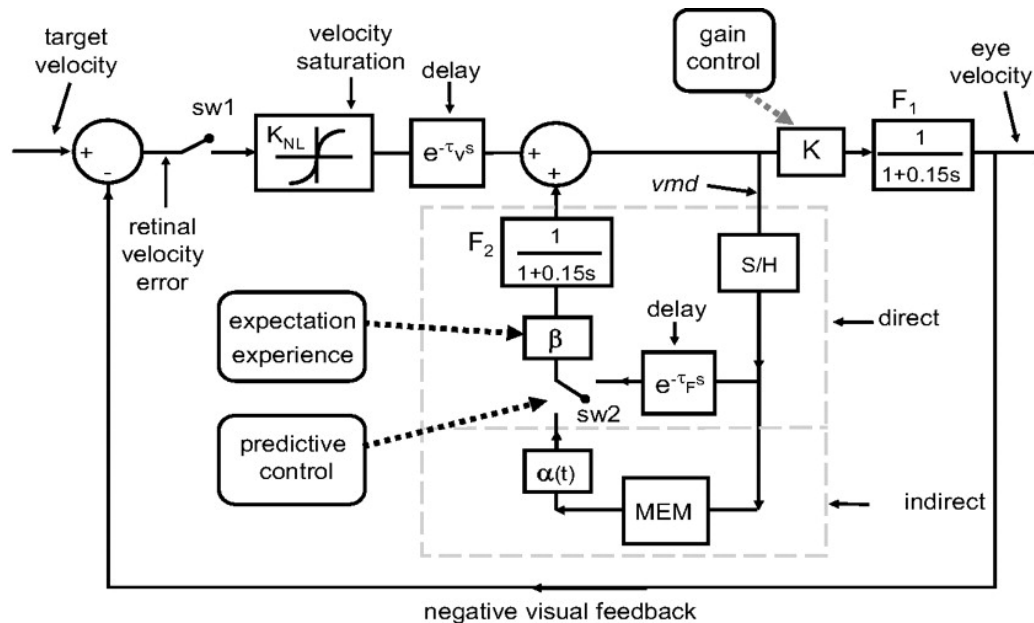
Blohm 2004



Adapted from Churchland et al. 2003

# Eye movements

- ▶ Smooth pursuit
  - ▶ Anticipatory pursuit

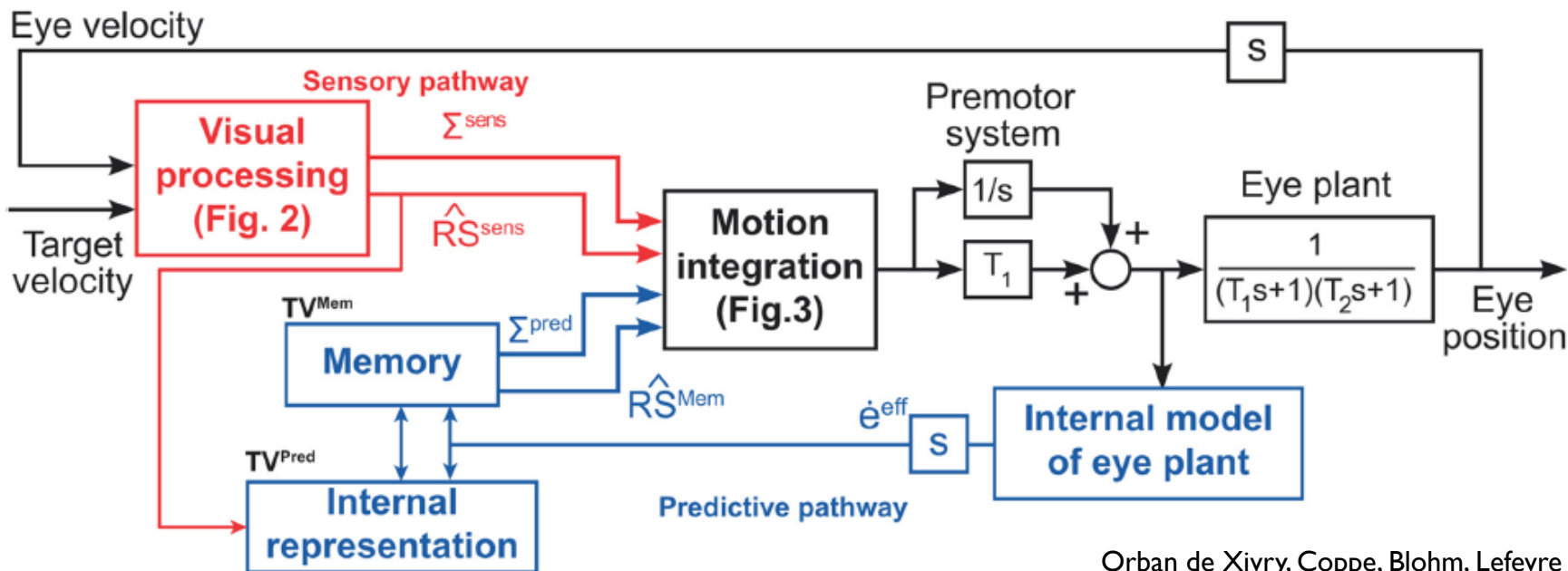
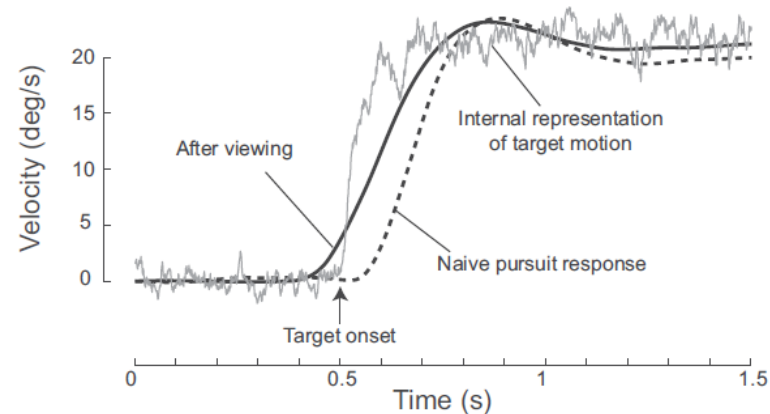


Barnes & Collins 2008



# Eye movements

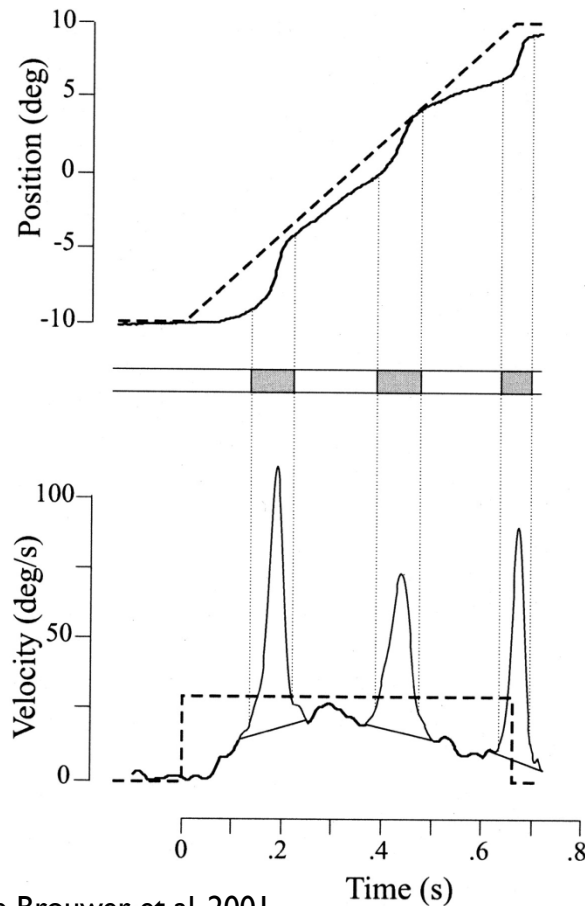
- ▶ Smooth pursuit
  - ▶ Stochastic pursuit model



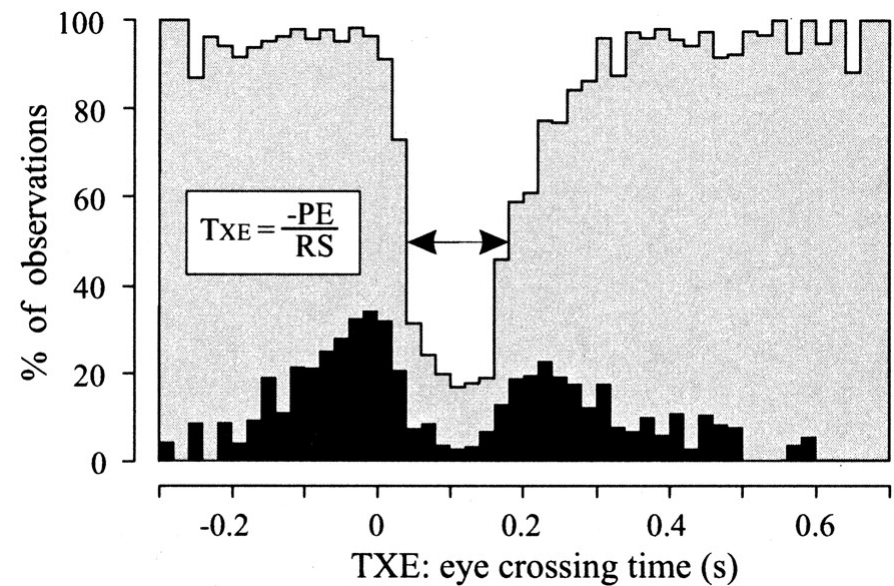
Orban de Xivry, Coppe, Blohm, Lefevre 2013

# Eye movements

## ► Saccade-pursuit interaction



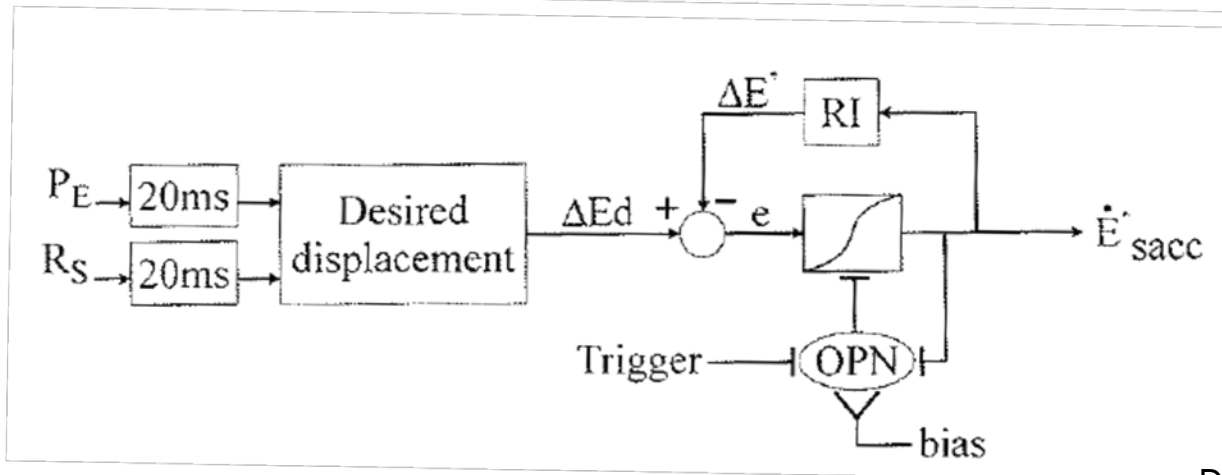
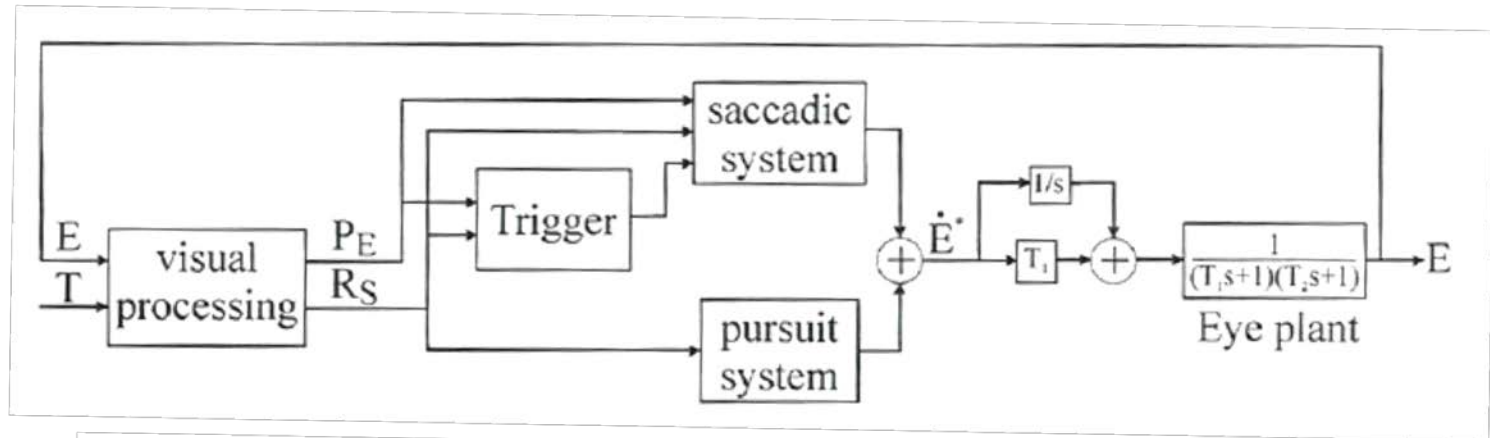
De Brouwer et al. 2001



De Brouwer et al. 2002

# Eye movements

## ► Saccade-pursuit interaction



De Brouwer et al. 2002



*That's all Folks!*