

# CoSMo

## Modeling Error Source Attribution During Motor Adaptation

By: EAGER BEAVeRS



(Elucidating Atribution Given Errors By Estimating A Vector Regression Solution)

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# 2018

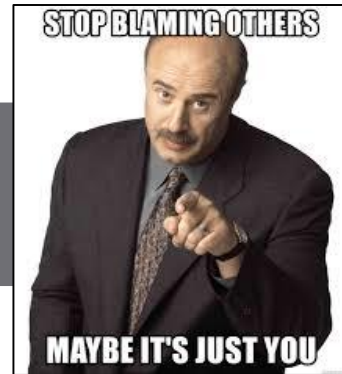
# Phenomenon: Adaptation



## Solution

Compensate for wind

Error probably due to **external** factors (the world)



Change muscle coordination, etc.

Error probably due to **internal** factors (the body)

*\*How do you identify the source of the error?\**

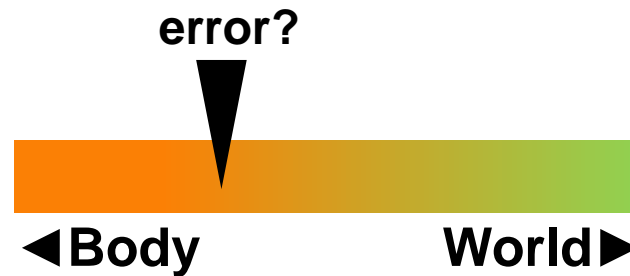


# Phenomenon, Cont'd

Large errors = attributed to world

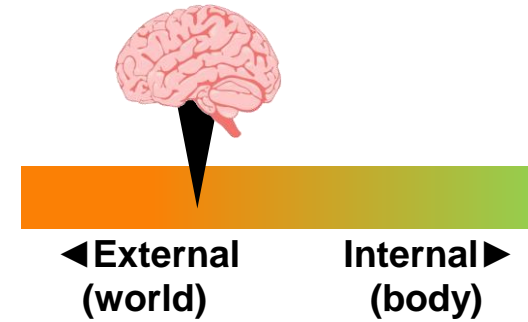
Small errors = attributed to body

*Wilke et. al, 2013*



# Question

*How is **motor error** attributed to the **body** and the **world**?*



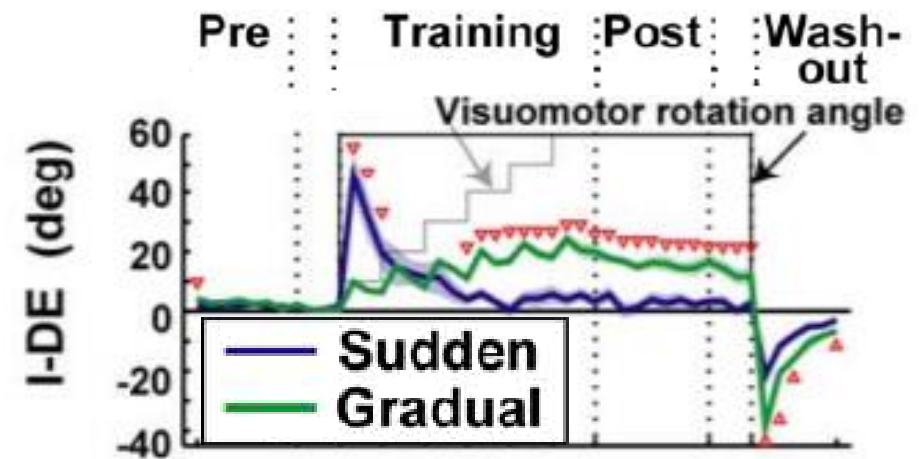
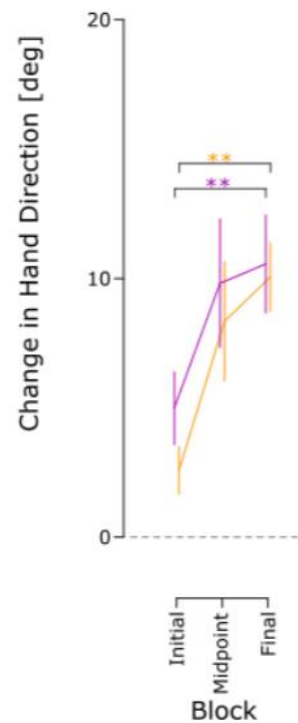
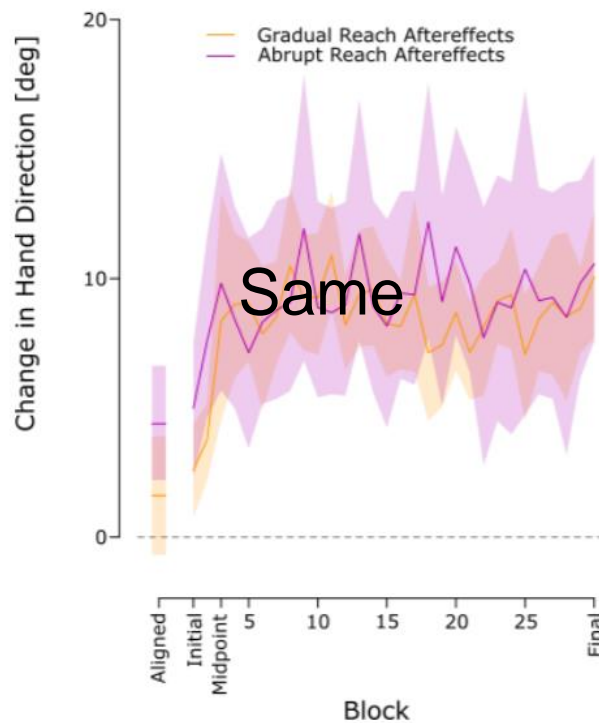
# Objectives

- Model how sensory prediction errors are **attributed to body and world** factors



# Background Motivation

- Current models **cannot** predict source attribution from sensory prediction error alone
- Conflicting experimental results for small and large errors



Saijo & Gomi., 2010

Different



# Background, Cont'd

## Estimating the sources of motor errors for adaptation and generalization

nature  
neuroscience

Max Berniker<sup>1,2</sup> & Konrad Kording<sup>1,2</sup>

*Constant World/  
Body Factors*



*Internal Model  
Parameters*



*Arm  
Kinematics*

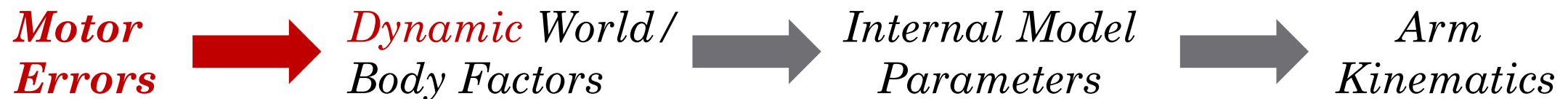


# Background, Cont'd

## Estimating the sources of motor errors for adaptation and generalization

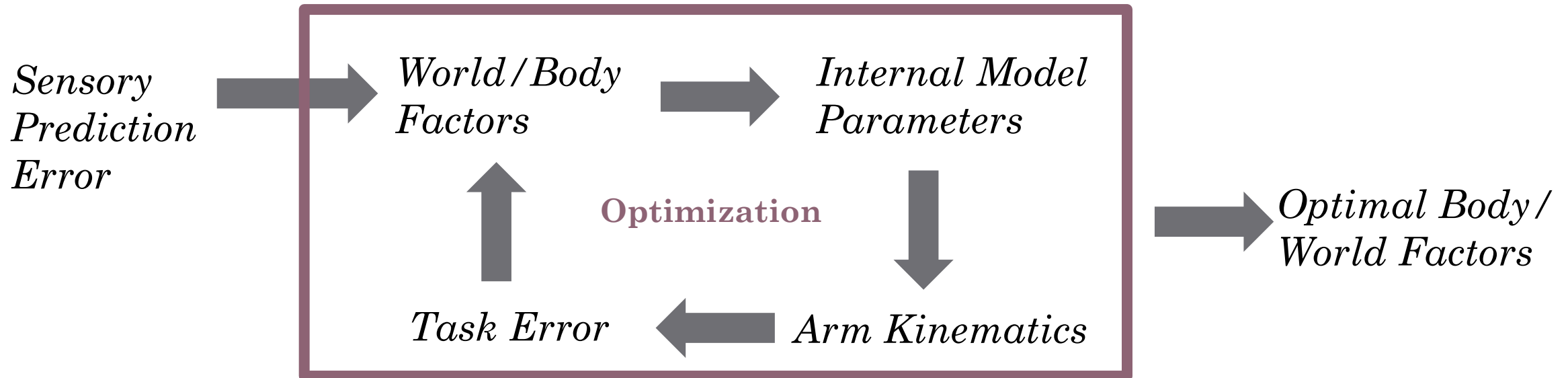
nature  
neuroscience

Max Berniker<sup>1,2</sup> & Konrad Kording<sup>1,2</sup>



# Project Overview

## Error Attribution Model



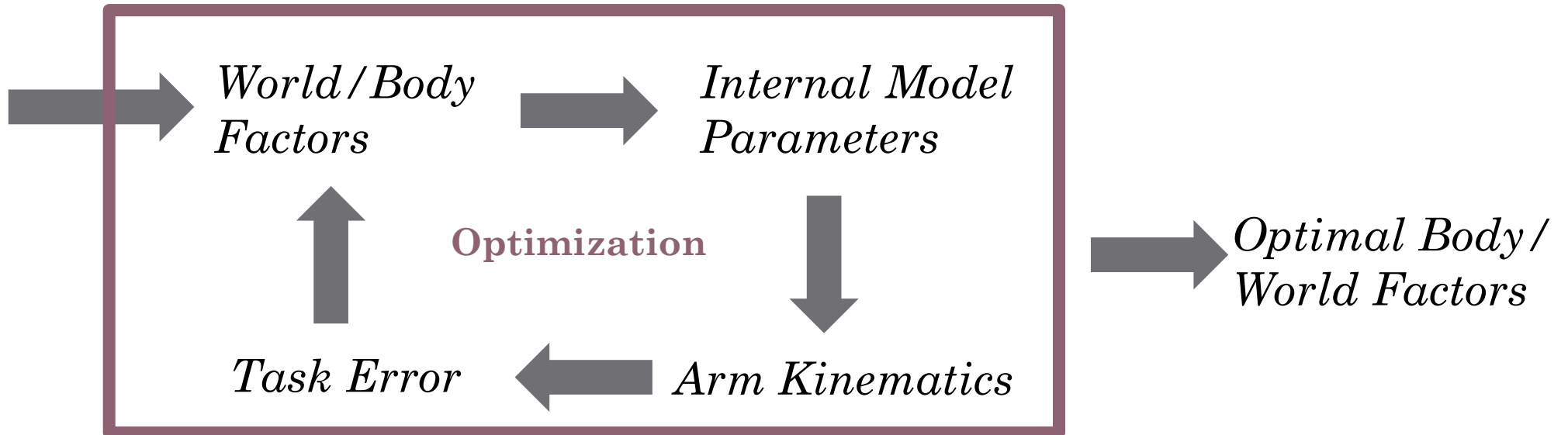


# Parameters & Variables

## Inputs

### Error Attribution Model

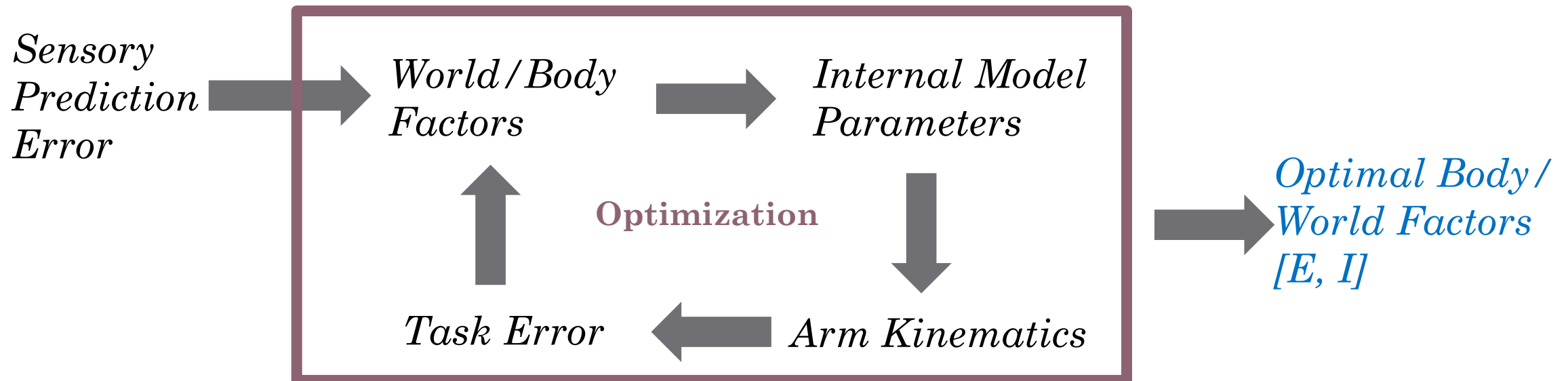
*Error*  
 $(SPE, \dot{SPE})$



# Parameters & Variables

Outputs

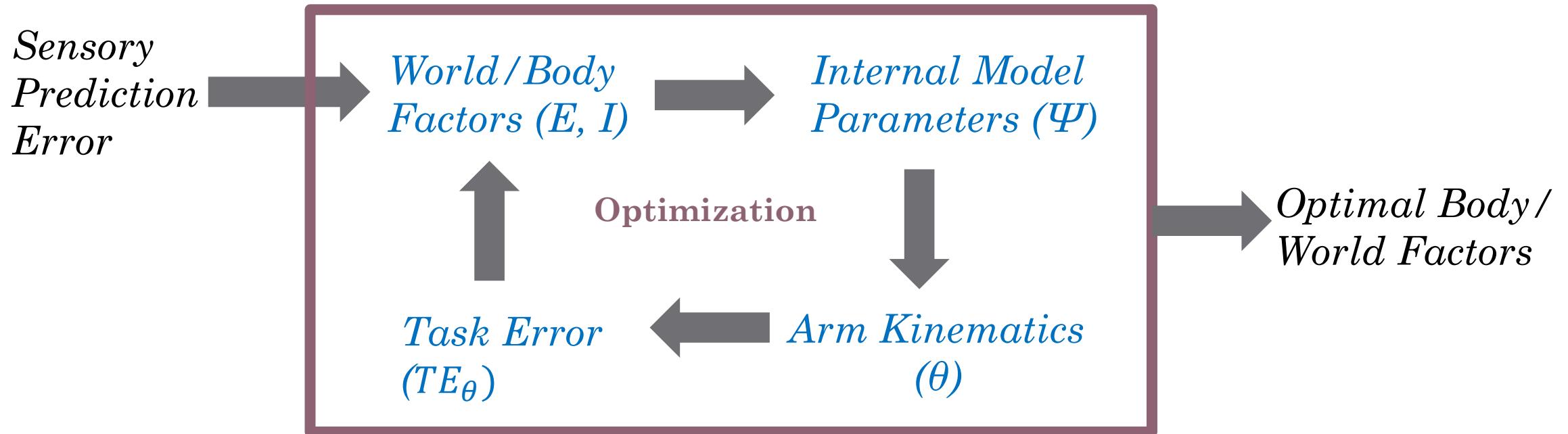
## Error Attribution Model



# Parameters & Variables

## Latent variables

### Error Attribution Model



# Parameters & Variables

## Error Attribution Model

Created by Us:

$$f(SPE) = \text{Error Attribution } [E, I]$$

### *Inputs*

*SPE*: Sensory Prediction Error

*SPE*: Rate of Sensory Prediction Error

### *Outputs*

*I*: Level of attribution to body (internal)

*E*: Level of attribution to world (external)

## Adaptation Component

Modified:

Internal Model  
Parameter Updates

Borrowed:

State  
Estimation

*Berniker & Kording, 2008*

### *Internal Model Parameters* [ $\psi$ ]

#### *Body*

- Arm Inertia
- Joint Damping

#### *World*

- External Forces on the Hand

### *States* [ $\theta$ ]

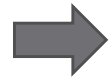
- Shoulder & Elbow Joint Angles
- Shoulder & Elbow Joint Angular Velocities



# Hypothesis

## *Hypothesis 1:*

**Small** external  
perturbation

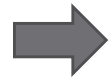


**small** initial  
 $SPE, \dot{SPE}$

**Small** errors will change the variance around the estimates of **body-centric** internal model parameters

## *Hypothesis 2:*

**Large** external  
perturbation



**large** initial  
 $SPE, \dot{SPE}$

**Large** errors will change the variance around the estimates of **world-centric** internal model parameters



# Simulated Experimental Paradigm

## *Center-out Reaching Task in Force Field*

**Assume humans use minimum jerk trajectory**

- Simulate path with no adaptation using **small** and **large** viscous force fields ( $F_{\perp hand}$ )
- Extract  $SPE$  ( $y - \hat{y}$ ) and  $S\dot{P}E$  ( $y - \dot{\hat{y}}$ ) to use in regression for finding  $[E, I]$

$$F_{\perp hand} \propto \|v_{hand}\|$$



# Toolkit

1. Regression
2. Global optimization
3. Kalman filtering



# Toolkit: Model Fitting & Optimization

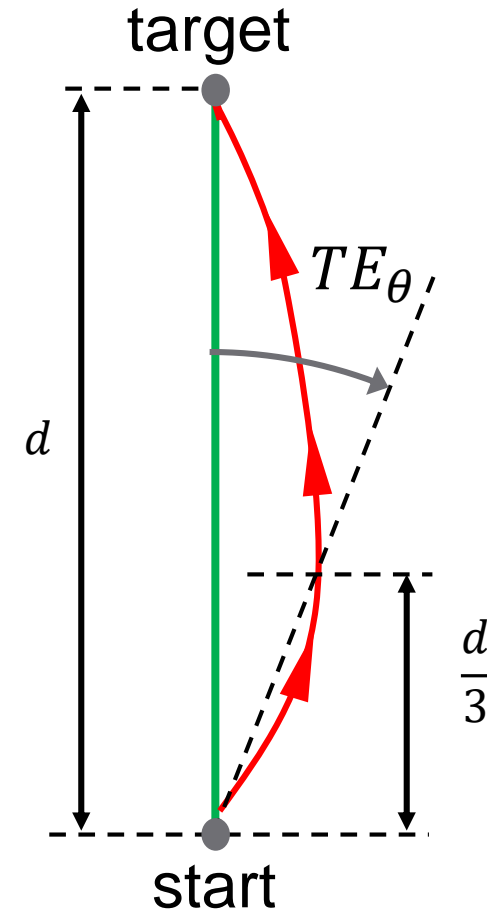
## *Estimation of Source Attribution Levels*

### 1. Regression

$$I = |a_1 SPE + a_2|$$

$$E = |b_1 SPE|$$

### 2. Optimize: Find $E$ & $I$ that Minimize Steady State Target Error, $TE_\theta$



*Step into adaptation model* →





# Toolkit: Kalman Filter

## *Adaptation: Internal Model Parameter Update*

At time  
step 1:

$[I, E]_1$



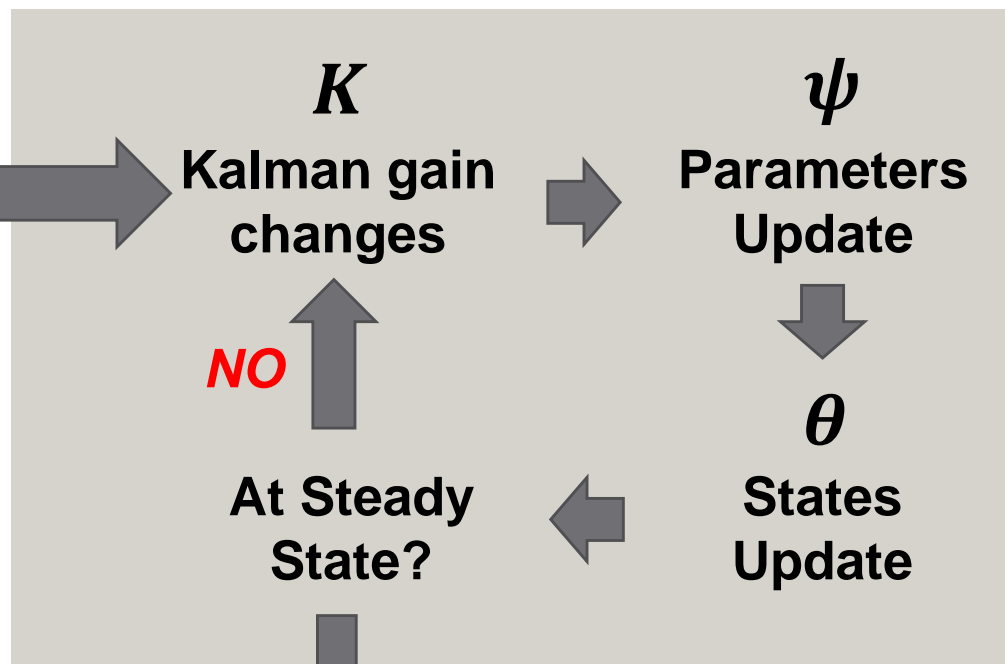
$Q_\psi =$

$$\begin{bmatrix} \text{oval}(\propto I_k^2) & 0 \\ 0 & \text{oval}(\propto E_k^2) \end{bmatrix}$$

Set initial  
covariance matrix



Step through trials  
until steady state  $TE_{\theta_k}$



YES

$TE_{\theta_k}$

*Step Out  
To Optimizer* →



# Predictions

Prediction: Increased SPEs will be attributed to the world  
and small SPEs will be attributed to the body

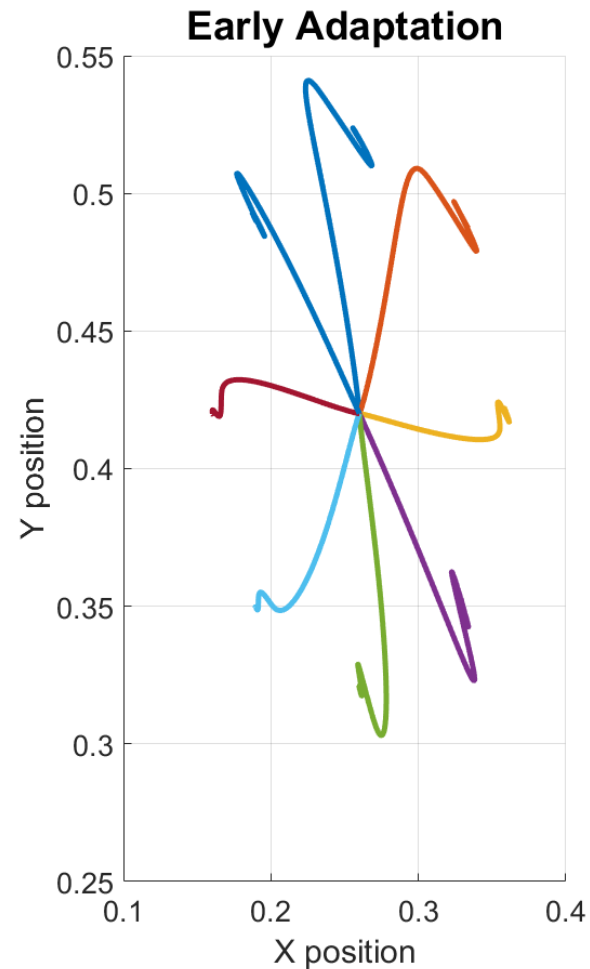
$$\frac{E}{I} = \textit{Source attribution}$$

Increased E/I = world attribution

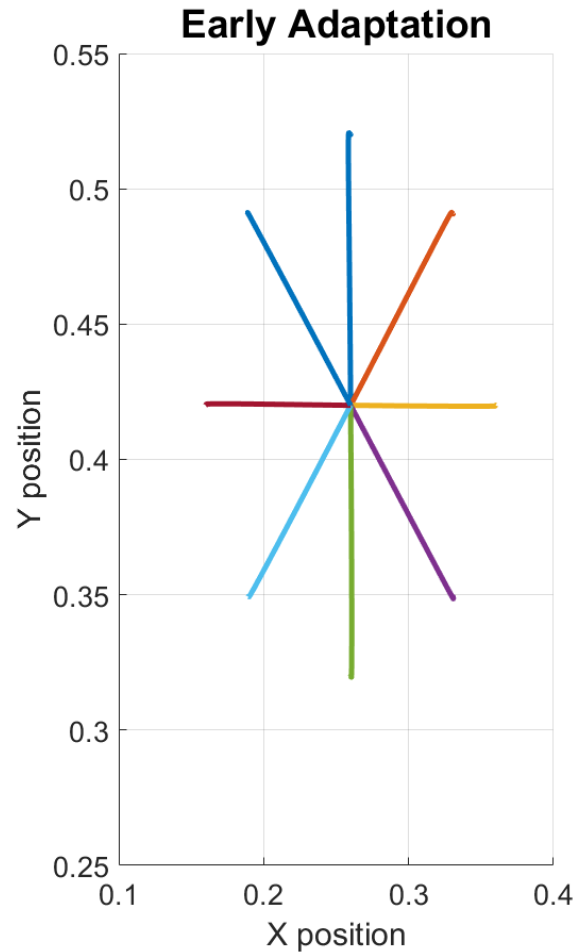
Decreased E/I = body attribution



# Results: Optimization yields improved learning parameters for large errors



# Results: Optimization yields poor learning parameters for small errors



# Results: E and I values change in expected directions

	E	I	E/I	Min Cost
Initial	.040	.010	.400	N/A
Large	.017	.035	.480	.161
Small	.0005	.044	.010	.210

1. Small errors will increase the variance around body-centric internal model parameter estimates (relative to world-centric variance)
2. Large errors will increase the variance around world-centric internal model parameter estimates (relative to body-centric variance)



# Model Testing

## *Hypothesis Revisited*

*How is **motor error** attributed to the **body**  
and the **world**?*

1. Large SPEs are attributed to the world
2. Small SPEs are attributed to the body



# Critical Model Evaluation

## Strengths of our model:

- Conceptually simple
  - Few additional parameters
- Interpretable
  - Change in model parameters directly relate to changes in real world parameters
- Makes predictions about how representation in an internal model
  - Useful for model validation



# Future Directions

- Improve our model's learning of small errors
  - Optimize for more parameters (rate of SPE, rate of task error)
- Validate model by comparing to experimental data
- Consider biological plausibility of our model





# Summary & Conclusions

- The body may differentially attribute movement errors to the body or world based on the magnitude of the SPE
  - Large errors attributed to the world
  - Small errors attributed to the body



**COSMOPROJECTON**  
GUNNAR BLOHM  
**52 WAYS TO MODEL YOUR IDEA**  
**BADS: L-Q-G... GOODBYE, CTRL+C!**  
KONRAD'S **TOP 10** ways to avoid Perfectly Pointless Publications

**2018**  
**EXPOSED! HARK Narks Q&A**  
Paul Schrater's latent variables revealed!  
**BRACE YOURSELF**  
**+Evolution's favorite tricks for faithful neuron encoding**

The image shows the cover of 'CosmoProjecton' magazine. The main cover image is of Gunnar Blohm, a man with short brown hair and a goatee, wearing a white shirt and a brown leather vest. To his right is a circular inset photo of Paul Schrater, a man with glasses and a goatee. In the bottom left corner, there is a smaller photo of Konrad, a man with glasses and a goatee, wearing a dark jacket and holding a small object. The background is a bright blue color with various text elements in yellow, white, and pink.

*Many thanks for guidance from  
Konrad, Paul, & Gunnar*

**Thank you for listening!  
Questions?**



# Equations

*Parameter Updates: Dynamics*

**Parameter Dynamics:**

$$\boldsymbol{\psi}_{k+1} = \alpha \mathbf{I} \boldsymbol{\psi}_k + \boldsymbol{\varepsilon}$$

$\alpha$  is forgetting factor

$\boldsymbol{\varepsilon} \sim N(\mathbf{0}, \mathbf{Q}_\theta) \leftarrow \text{fictitious}$

**Fictitious Observer:**

$$\boldsymbol{\varphi}_k = \begin{bmatrix} \frac{\partial e}{\partial \psi} \end{bmatrix} \vec{e}$$

$\vec{e} \sim N(\mathbf{0}, \mathbf{R}_\varphi)$

$\text{Jacobian} = H$



# Equations

**Prior Covariance:**

$$\begin{aligned}\mathbf{P}_{k+1|k} &= \text{COV}(\boldsymbol{\Psi}_{k+1|k}) \\ &= \mathbf{A}^T \mathbf{P}_{k|k} \mathbf{A} + \mathbf{Q}_\theta\end{aligned}$$

**Kalman Gain:**

$$\mathbf{K}_{k+1} = \frac{\mathbf{P}_{k+1|k} \mathbf{H}^T}{\mathbf{R}_\varphi + \mathbf{H} \mathbf{P}_{k+1|k} \mathbf{H}^T}$$

**Update:**

$$\text{param}_{k+1} = \text{param}_k + k_{k+1} \overbrace{(y - \hat{y})}^{\text{SPE}}$$

**Observation Covariance:**

$$\mathbf{R}_\varphi = \text{COV}(\boldsymbol{\varphi}_k)$$

**Posterior Covariance:**

$$\mathbf{R}_{k+1|k+1} = (\mathbf{P}^{-1} + \mathbf{H}' \mathbf{R} \mathbf{H})^{-1}$$



# Project Overview

## Error Attribution Model

$SPE, \dot{SPE}$   
From perturbed  
reaching task

**Goal: minimize target error**  
*Guess attribution to each of  
body & world factors,  
estimate associated task error*

**Adaptation Model Component**  
Simulate adaptation to perturbed  
reaching task until steady state &  
output target error

*Keep guessing until  $I$  &  $E$  that  
minimize error found*

$I$  Level of error  
attribution to body  
(internal)

$E$  Level of error  
attribution to world  
(external)

